Inelastic Analysis — Plastic Analysis

- ELASTIC ANALYSIS IS VALID FOR
  – Small Displacements &/or
  – Linear material Properties i.e stress-strain relationship remains linear

- NON-LINEAR PROBLEM

  Geometrical Nonlinear Problem --- If the displacements are not small i.e. large displacements problems.

  Material Nonlinear/Inelastic/Plastic Problem --- the stress strain relationships of the material is non-linear

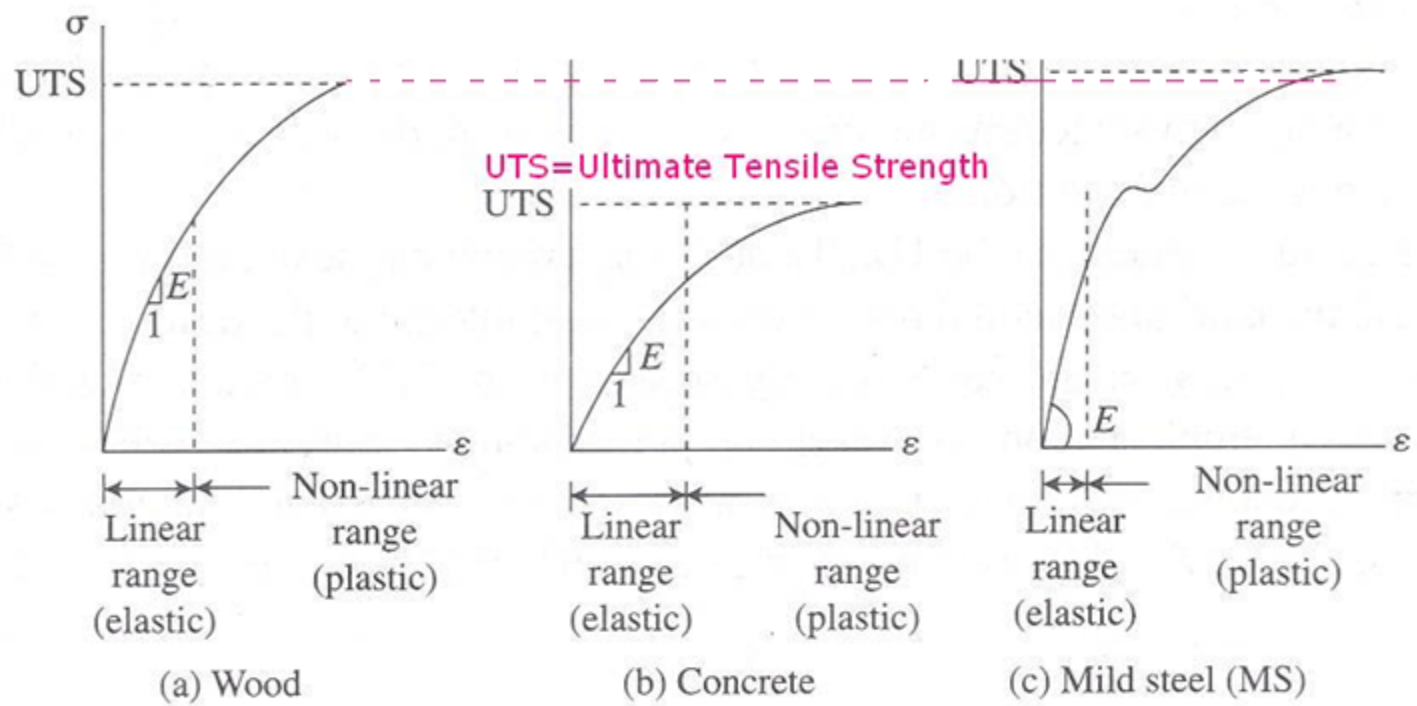
- A Problem can be both Geometrically & Materially Non-linear.
- Accordingly, in the literature the terminology like
  - LARGE DISPLACEMENT & SMALL STRAIN PROBLEM,
  - SMALL DISPLACEMENT & LARGE STRAIN PROBLEM
  - LARGE STRIAN & LARGE DISPLACEMENT PROBLEMS

  are mentioned.

- LARGE DISPLACEMENT& SMALL STRAIN PROBLEM
  - This deals with **Geometrically Nonlinear problems**. For this category, strain-displacements relationships are **nonlinear** but stress-strain relationships are **linear.**

- SMALL DISPLACEMENT & LARGE STRAIN PROBLEMS
  - This is equivalent to **Material Nonlinear Problems** where the **large strain** along with **material nonlinearity** is governing criterion for **Nonlinearity** in the system
- LARGE STRIAN & LARGE DISPLACEMENT PROBLEMS
  - Both material nonlinearty & nonlinear relationship of strain and displacement is involved.

- Linear and Nonlinear Structures — Material Nonlinearity
  - Different materials possesses different properties, load resistance and deformation characteristics.



Typical stress–strain relation of various materials.

- As the initial slope of the curve is different for these material

  which ⇒ E is different

  We Know that EI directly governs the displacements of the Structure

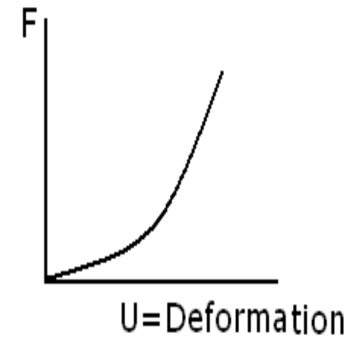
  - Linear Structure— when the stresses developed are within **Elastic Limit**
  - **Non-linear Structure** — Stresses developed are in the plastic range or non-linear range.
  - **Several factors influence the stress-strain properties of a material including the loading rate(load history) and the duration of the load, environmental condition.**

- Geometrical Nonlinearity— In addition to material nonlinearity, some structures may exhibit non-linear characteristics in its overall behavior due to change in its shape under loading by undergoing displacements by a significant amount (large displacements) to maintain its overall equilibrium.

  Examples are

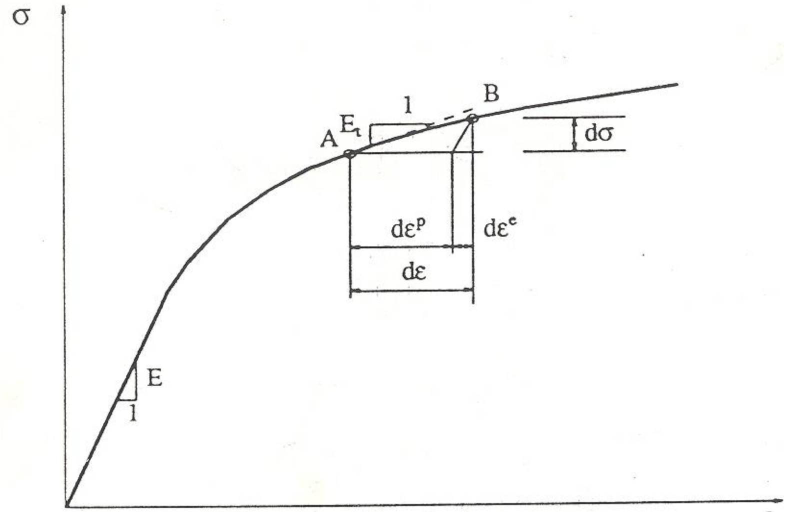
  (i) deformation of cable structures

  (ii) deformation of Pole Vault



- For nonlinear problems
  - No one to one correlation between stress (σ) and strain ε exists
  - Not possible to express the stress-strain relationship in terms of total stress and total strain

And hence, for elastic-plastic materials, only a unique incremental relationship between stress and strain increments can be written and expressed in terms of stress and deformation history



Stress and strain increments

- The strain increment $d\varepsilon$ is decomposed into two parts: the elastic strain increment $d\varepsilon^e$ and the plastic strain increment $d\varepsilon^p$ i.e.

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \qquad (1)$$

$$d\sigma = E_t\, d\varepsilon = E\, d\varepsilon^e = E_p\, d\varepsilon^p \qquad (2)$$

- Where $d\sigma$ is the corresponding stress increment, E the Youngo's modulus, E_t and E_p the tangential modulus and plastic modulus respectively. E, E_t and E_p may be derived from an experimental stress-strain curve under a monotonic loading condition. As given below:

Note that

$$E_t = \frac{d\sigma}{d\varepsilon}, \quad E_p = \frac{d\sigma}{d\varepsilon^p} \tag{3}$$

$$E_t = \frac{d\sigma}{d\varepsilon}$$

$$\Rightarrow \frac{1}{E_t} = \frac{d\varepsilon}{d\sigma} = \frac{d\varepsilon^e + d\varepsilon^p}{d\sigma}$$

$$\frac{1}{E_t} = \frac{d\varepsilon^e}{d\sigma} + \frac{d\varepsilon^p}{d\sigma}$$

$$\frac{1}{E_t} = \frac{1}{E} + \frac{1}{E_p}$$

from which, we obtain the relation

$$\frac{1}{E_t} = \frac{1}{E} + \frac{1}{E_p} \tag{4}$$

or

$$E_t = \frac{E\,E_p}{E + E_p}, \quad E_p = \frac{E\,E_t}{E - E_t} = \frac{E_t}{1 - \frac{E_t}{E}} \tag{5}$$

- For a given elastic-plastic state, a stress increment or a strain increment may cause PLASTIC LOADING or elastic UNLOADING.
- In case of plastic loading, new plastic deformation accumulates. In case of elastic unloading, no new plastic deformation occurs.
- For obtaining the solution of an elastic-plastic problem, the actual elastic-plastic material behaviour must be idealised. For 1-D probles, the elastic plastic behaviour can be represented by idealised stress-strain relations together with an assumed hradening rule.

PLASTIC ANALYSIS

Working stress method: is based on the *working loads*. At *Working loads* the stress distribution (Both in steel & Conc) is assumed linear and **design** is based on assuming the linear stress strain relationships ensuring that the stresses both in conc. & steel do not exceed σ_{y} at service loads.

$$\Rightarrow \text{service load/working load} = \frac{\text{ultimate or yield strength of material}}{\text{factor of safety}}$$

Or factor of safety is the ratio of *ultimate load* and *working* or service load

Service Load & Working loads are one and same

The load for which the structure is considered to be designed.

Elastic Theory

(1) Stress-strain relation assumed to be linear

(2) Structure fails if the stress at the maximum stressed point reaches yield stress

(3) The *service load* is restricted to the value such that corresponding to maximum stressed point, the stress is equal to the *working stress*

(4) *i.e.* the structure can take working loads.

- In the elastic analysis it is assumed that the structure would fail if the design load is applied the factor of safety times.
- An elastic analysis of structure is important to study the performances, especially with rgard to serviceability, under the service loading for which the structure is designed.

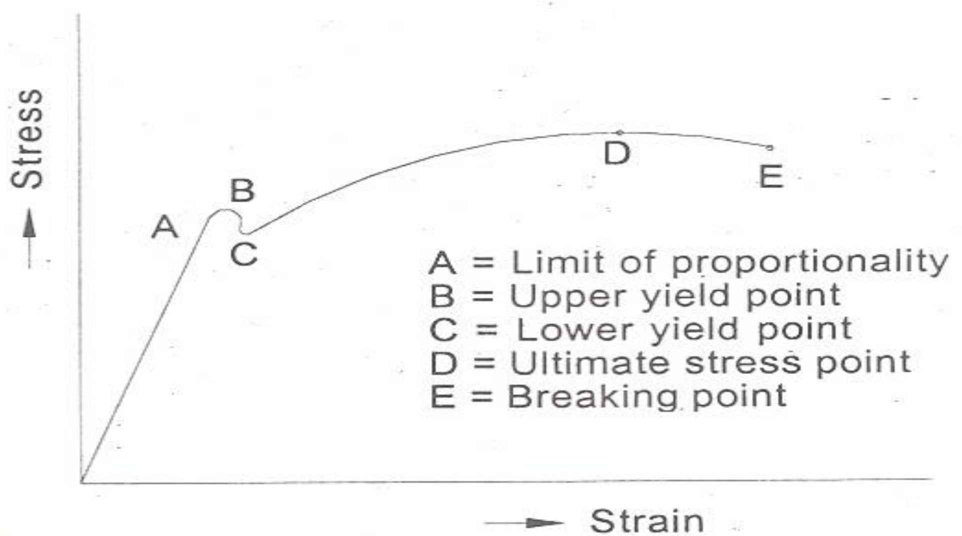
Disadvantages:

- Does not provide a uniform overload capacity for all sections of members & hence the need of ultimate strength theory emerged

- Plastic design is based on the philosophy of failure of member or srtucture rather than their condition at working stress or service loads.
- A member is designed employing the criteria that the structure will fail at a load substantially higher than the working or service loads.

INTRIDUCTION TO PLASTIC ANALYSIS

The stress-strain curve is linear between the origin and the elastic limit, which is very close to the yield point; After the upper yield point, there is a sudden drop in stress to lower yield point. The designer normally treats the lower yield point as the limit of proportionality. From this yield point to the ultimate stress point.

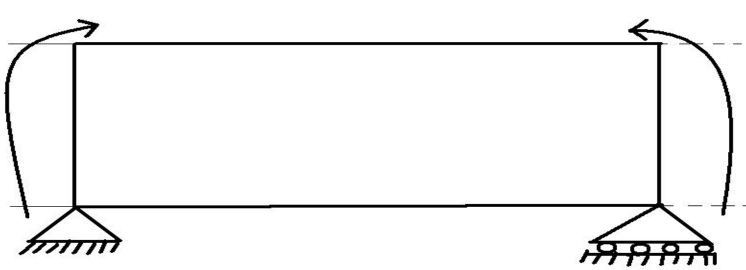


- the zone is called *strain hardening zone.* At ultimate stress point, neck formation starts and the load carrying capacity reduces. Finally, breaking takes place at stress (normal stress) which is less than the ultimate stress.

Rationale for Plastic Analysis

Now consider stresses across the highly stressed section of the simply supported beam as load increases

- We take for example an arbitrary section shown in Fig. (a). For small deformation, when the bending stresses are small and within the elastic range, i.e., ($\sigma < \sigma_y$ the stress distribution is linear across the section as in stage I in Fig. (b). In this case, the NA will pass through the centroid of the section. As the moment is further increased, stresses in either of the extreme fibres reach yield value shown in stage II in Fig. (b), with NA still passing through the centroid. The value of the moment corresponding to this yield is called the *yield moment, M_y* . As the moment increases further, the bottom fibre also yields and yielding at top fibre progresses inwardly shown in stage III in Fig. (b). In this case, the NA is shifted below the centroidal axis to satisfy the equilibrium requirements and is determined from the consideration of the total compressive force equal to the total tensile force over the cross section.



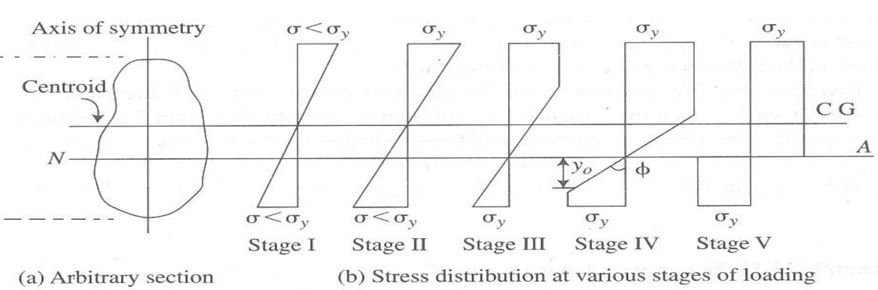
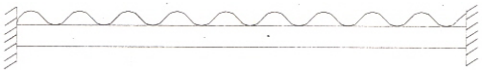


Fig. 23.17 Bending of beam.

- The yielding progresses inwardly from both top and bottom fibres towards NA with further increase in moment shown in stage IV in (b). When the load reaches its ultimate value, the yield progresses right up to the NA and the entire section becomes *fully plastic* as in stage V in Fig. (b). The moment corresponding to this stage is called ***fully plastic moment*, M_p**.
- If we neglect the strain hardening in the outer fibres, there cannot be any further increase in moment. Therefore, the plastic moment represents the limiting strength of the beam in bending. The NA in the case of fully plastic section will pass through the axis of equal areas. If both axes of a section are symmetrical, then the locations of NA in elastic and fully plastic conditions remain unchanged.
- When the fully plastic moment is reached, the section will act as a hinge permitting rotation. The yield will spread in the longitudinal direction with further increase in load.

- Now, let us consider the load carrying capacity of a fixed beam. As the bending moment is maximum at supports, first extreme fibres at supports yield. For further increase of load, entire section at supports yields. Even at this stage, the structure will not collapse, since a beam with two hinges at ends is a stable structure. For further load, it acts as a simply supported beam till all fibres at the mid-span section yield.



Hence one can say that

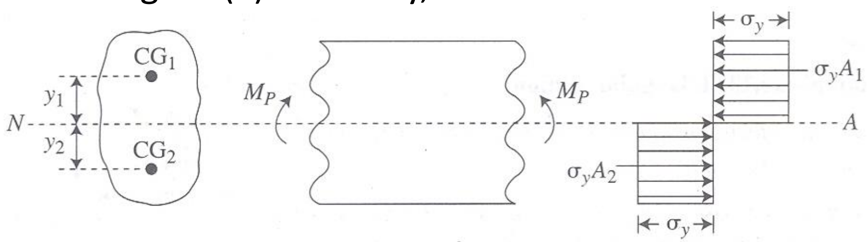
- the elastic theory under estimates the load carrying capacity of the structure. For indeterminate structures, this under estimation is still high.
- not giving the correct idea about the load carrying capacity of the structures.

PLASTIC MOMENT OF RESISTANCE

- The moment of resistance developed by a fully plastic section is called the ***fully plastic moment, Mp***. For the evaluation of the fully plastic moment, we make the following **assumptions**:

1. The material is homogeneous and isotropic in the elastic as well as in the plastic states.
2. Hooke's law is applicable in the elastic stage of the material. In the plastic stage, the stress remains constant.
3. The yield stress and the modulus of elasticity have the same values both in compression and in tension.
4. Plane sections remain plane both before and after bending.
5. No resultant axial force exists on the beam.
6. The cross section of the beam is Symmetrical about an axis which passes through the centroid of the beam as well as parallel to the plane of bending.
7. Every fibre of the beam can stretch and shorten under stress both longitudinally and laterally without any restraint from other layers.

- Let us now consider a cross section of a beam shown in Fig.(a). We apply a fully plastic moment *Mp* of sagging nature on the beam shown in (b). Due to the application of moment *Mp* every fibre of the cross section is stressed to the yield level of σ_y and the corresponding stress distribution is rectangular as in Fig. (c). The nature of stress in fibres above the NA is compressive and that below is tensile. We denote the area of the upper portion of the cross section as A_1 and the distance of its CG from the NA as y_1 shown in Figure (a). Similarly, we denote the area of the lower portion as A_2 and its CG distance Y_2 .



(a) Cross-section (b) Application of moment (c) Stress distribution at plastic stage

Plastic moment of resistance.

- The compressive force acting on the upper portion of the cross section, $C = \sigma_y A_l$ .The tensile force acting on the lower portion, $T = \sigma_y A_2$. From equilibrium consideration, C = *T,* i.e., *i.e.* $\sigma_y A_l = \sigma_y A_2$ or $A_l = A_2$,

  However, total area $A = A_l + A_2$, $A_l = A_2 = A/2$.
- Therefore, the neutral axis (NA) divides the cross section into two equal parts.
- As the CG of compressive and tensile forces lie at a distance from the NA, they would give rise to a couple which has to be equal to the externally applied moment, M_p. Therefore, taking moment about NA, we get $\sigma_y A_l y_1 + \sigma_y A_2 y_2 = Mp$. We know $A_l = A_2 = A/2$. So,

$$M_p = \sigma_y \frac{A}{2}(y_1 + y_2) \qquad (1)$$

$$M_p = \sigma_y S \quad \text{where } S = \frac{A}{2}(y_1 + y_2) \qquad (2)$$

Equation (1), is the expression for the *plastic moment of resistance* of a section.

PLASTIC MODULUS

- In Eq. (1), the quantity $\frac{A}{2}(y_1 + y_2)$ is called the *plastic section modulus.* It is the sum of the moments of areas of the compression and tension zones about NA

Example 23.12 Rectangular section

Determine the plastic section modulus of a rectangular section shown in Fig. 23.19.

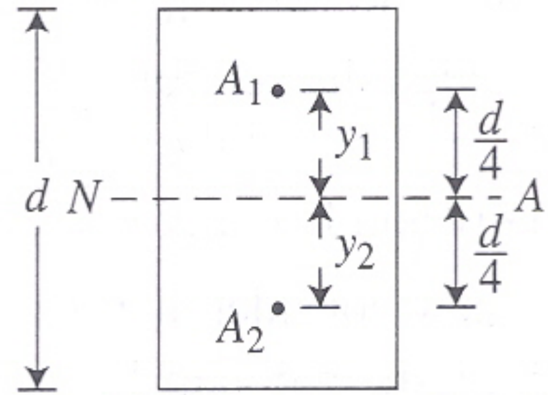


Fig. 23.19 Rectangular section.

Solution The breadth of the section is b and depth d. Area of upper zone is A_1 and lower zone A_2. Distance of CG of A_1 is y_1 and that of A_2 is y_2. However, it is regular and symmetrical

$$y_1 = y_2 = \frac{d}{4}; \quad A = bd$$

We know $S = \frac{A}{2}(y_1 + y_2) = \frac{bd}{2} = \left(\frac{d}{4} + \frac{d}{4}\right) = \frac{bd^2}{4}$ (23.9)

If $b = 150$ mm and $d = 300$ mm, then

$$S = \frac{150 \times 300^2}{4} = 33{,}75{,}000 \text{ mm}^3$$

Example 23.13 Triangular section

Find the plastic modulus of a triangular section shown in Fig. 23.20.

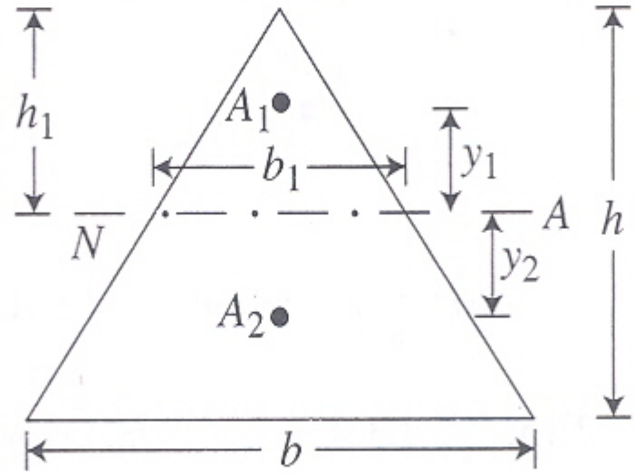


Fig. 23.20 Triangular section.

Solution The base width of the triangle is b and its height is h. Area of triangle, $A = bh/2$. Now, we have to divide the triangle into two zones of equal areas. Let us assume that the axis that divides the triangle into equal areas lie at a distance of h_1 from the apex. The width at that axis be b_1 (Fig. 23.20). Then

$$\frac{b_1 h_1}{2} = \frac{1}{2}\frac{bh}{2} \tag{1}$$

We know from Fig. 23.20 that

$$\frac{h_1}{h} = \frac{b_1}{b} \quad \text{or} \quad b_1 = \frac{b h_1}{h} \tag{2}$$

Substituting Eq. (2) in Eq. (1), we get

$$\frac{b_1 h_1}{h}\frac{h_1}{2} = \frac{1}{2}\frac{bh}{2} \qquad \therefore \qquad h_1 = \frac{h}{\sqrt{2}} \tag{3}$$

Substituting Eq. (3) in Eq. (2), we get $b_1 = b/\sqrt{2}$.

$$\text{Now, } y_1 = \frac{h_1}{3} = \frac{h}{3\sqrt{2}} = 0.235\,h \tag{4}$$

$$\text{and} \quad y_2 = \frac{(h - h_1)}{3} \times \frac{(b_1 + 2b)}{b_1 + b} = \frac{\left(h - \dfrac{h}{\sqrt{2}}\right)}{3} \times \frac{\left(\dfrac{b}{\sqrt{2}} + 2b\right)}{\left(\dfrac{b}{\sqrt{2}} + b\right)}$$

$$= \frac{\left(8 - 5\sqrt{2}\right)h}{6} = 0.155h$$

Therefore, plastic modulus

$$S = \frac{A}{2}(y_1 + y_2) = \frac{1}{2}\frac{bh}{2}(0.235h + 0.155h) = 0.098bh^2 \tag{23.10}$$

If $b = 75$ mm and $h = 100$ mm, then $S = 0.098 \times 75 \times 100^2 = 73{,}500\ \text{mm}^3$.

Example 23.14 Circular section

Determine the plastic modulus of a circular section of diameter d as shown in Fig. 23.21.

Solution

Area of circle $= \frac{\pi}{4}d^2.$

We know that the CG of the semicircle from NA $= 2d/3\pi$.

$$\therefore \quad y_1 = y_2 = \frac{2d}{3\pi}$$

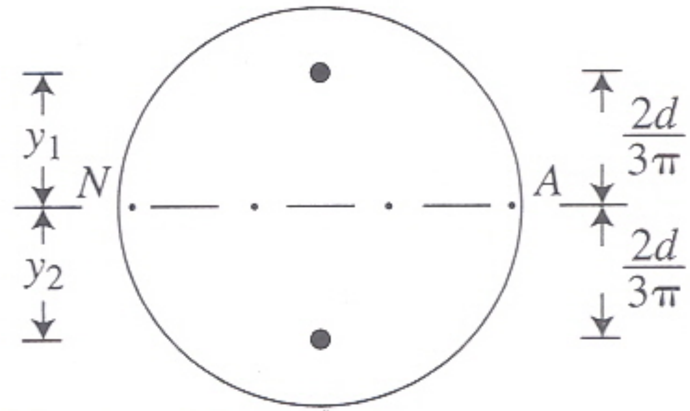


Fig. 23.21 Circular section.

Plastic modulus $S = \frac{A}{2}(y_1 + y_2)$

$$= \frac{1}{2} \times \frac{\pi}{4}d^2\left(\frac{2d}{3\pi} + \frac{2d}{3\pi}\right) = \frac{d^3}{6} \tag{23.11}$$

If the diameter of the circle is 125 mm, then

$$S = \frac{125^3}{6} = 3{,}25{,}520.83\,\text{mm}^3$$

Example 23.15 I section

Determine the plastic modulus of the I section shown in Fig. 23.22.

Solution The given cross-section is symmetrical. Therefore, NA lies at centroid of the section. We determine the plastic modulus of the section by taking moment of areas of individual rectangles about NA.

$$S = 2\left[0.5d \times 0.1d \times \left(\frac{d}{2} - \frac{0.1d}{2}\right) + \left(\frac{d}{2} - 0.1d\right) \times 0.1d \times \frac{1}{2}\left(\frac{d}{2} - 0.1d\right)\right]$$

$$= 2[0.0225d^3 + 0.008d^3] = 0.061d^3$$

If $d = 450$ mm, then $S = 0.061 \times 450^3 = 55{,}58{,}625\ \text{mm}^3$.

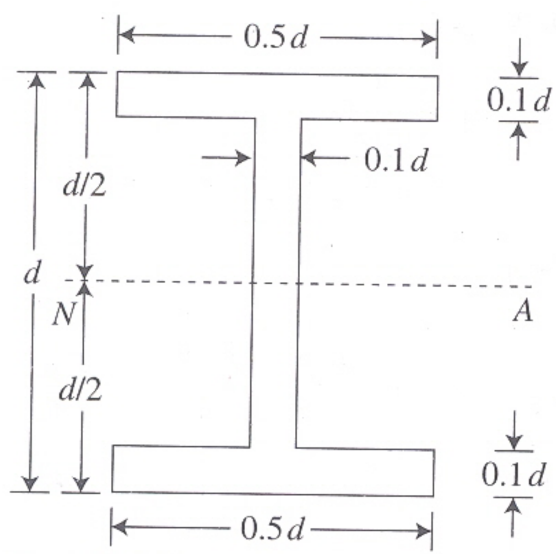


Fig. 23.22 I section.

SHAPE FACTORS FOR VARIOUS SECTIONS

- We know, that the ratio of the moment of inertia about the NA of a section to the distance of the extreme fibre is called the *section modulus*, **Z**. Let the moment of inertia be I and the distance of extreme fibre be y_c, Then

$$Z = \frac{I}{y_c}$$

The bending moment M is given by $M = \sigma Z$. Also, the yield moment M_y, i.e., the moment at which the first yield occurs, with the section still remaining elastic is given by $M_y = \sigma_y Z$. The plastic moment from Eq. (**2**) is $M_P = \sigma_y S$.
The ratio of plastic moment to the yield moment is called the *shape factor*, η

$$\therefore \quad \eta = \frac{M_P}{M_y} = \frac{\sigma_y S}{\sigma_y Z} = \frac{S}{Z} \qquad (3)$$

From this, it is clear that the shape factor also refers to the ratio of plastic modulus S to the section modulus Z. The shape factor is the property of a section and solely depends on the shape of the cross section. We now evaluate the shape factor for some well-known sections.

Example 23.16 Rectangular section

Determine the shape factor of a rectangular section of width b and depth d.

Solution The moment of inertia of the section

$$I = \frac{1}{12} bd^3$$

The distance of extreme fibre is

$$y_c = \frac{d}{2}$$

Section modulus, $Z = \frac{I}{y_c} = \frac{1}{12} bd^3 \times \frac{2}{d} = \frac{1}{6} bd^2$

From Example 23.12, plastic section modulus of rectangular section,

$$S = \frac{bd^2}{4}$$

$\therefore$ Shape factor, $\eta = S/Z = (1/4)bd^2 \times 6/(bd^2) = 1.5$

Example 23.17 Triangular section

Evaluate the shape factor of a triangular section with base width b and height h.

Solution The moment of inertia about centroidal axis, $I = \dfrac{bh^3}{36}$

Distance of extreme fibre from the centroid, $y_c = \dfrac{2}{3}h$

The section modulus, $Z = \dfrac{I}{y_c} = \dfrac{bh^3}{36} \times \dfrac{3}{2h} = \dfrac{bh^2}{24}$

We know from Example 23.13, plastic section modulus of triangular section is $S = 0.098\,bh^2$.

$$\therefore \text{Shape factor, } \eta = \frac{S}{Z} = \frac{0.098bh^2}{\left(\dfrac{bh^2}{24}\right)} = 2.34$$

Example 23.18 Circular section

Find the shape factor of a circular section of diameter, d.

Solution The moment of inertia of the section is

$$I = \frac{\pi d^4}{64}$$

Distance of extreme fibre from centroid

$$y_c = \frac{d}{2}$$

$$\therefore \text{ Section modulus, } Z = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

We know from Example 23.14, the plastic section modulus of the circular section is

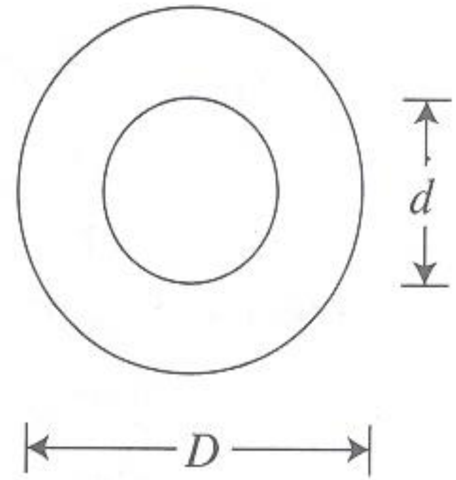
$$S = \frac{d^3}{6}$$

$$\therefore \text{ Shape factor, } \eta = \frac{S}{Z} = \frac{d^3}{6} \times \frac{32}{\pi d^3} = 1.7$$

Example 23.19 Hollow circular section

Find the shape factor for a hollow circular section with inner diameter d and outer diameter D shown in Fig. 23.23.

Solution



Hollow circular section

Let the diameter ratio be $\gamma = \dfrac{d}{D}$

Moment of inertia, $I = \dfrac{\pi}{64}(D^4 - d^4)$

Distance of extreme fibre, $y_c = \dfrac{D}{2}$

$\therefore$ Section modulus, $z = \dfrac{\pi}{64}(D^4 - d^4) \times \dfrac{2}{D}$

$$= \frac{\pi}{32D}[D^4 - (\gamma D)^4] = \frac{\pi}{32}D^3(1 - \gamma^4)$$

We know from Example 23.14, plastic section modulus of a solid circular section is $S = d^3/6$. Using this, we get for hollow circular section

$$S = \frac{D^3}{6} - \frac{d^3}{6} = \frac{D^3}{6} - \frac{(\gamma D)^3}{6} = \frac{D^3}{6}(1 - \gamma^3)$$

$$\therefore \text{ Shape factor, } \eta = \frac{S}{Z} = \frac{D^3}{6}(1 - \gamma^3) \times \frac{32}{\pi D^3} \times \frac{1}{(1 - \gamma^4)}$$

$$= \frac{16(1 - \gamma^3)}{\pi\,(1 - \gamma^4)}$$

$$= 1.7\frac{(1 - \gamma^3)}{(1 - \gamma^4)}$$

If $d = 50\,\text{mm}$ and $D = 75\,\text{mm}$, $\gamma = (50/75) = 0.67$.

$$\text{Then, } \eta = 1.7\frac{(1 - 0.67^3)}{(1 - 0.67^4)} = 1.49$$

Example 23.20 Unsymmetrical I section

Determine the shape factor of the I section shown in Fig. 23.24.

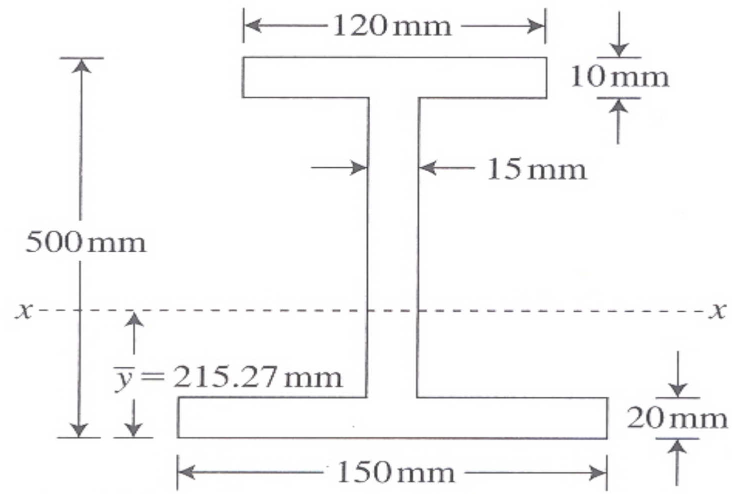


Fig. 23.24 Unsymmetrical I section.

Solution The given section is unsymmetrical. So, we have to determine the CG of the section. We take the base as reference and determine the distance of the CG from the base is

$$\bar{y} = \frac{150 \times 20 \times 10 + 470 \times 15 \times 255 + 120 \times 10 \times 495}{150 \times 20 + 470 \times 15 + 120 \times 10}$$

$$= \frac{30000 + 1797750 + 594000}{3000 + 7050 + 1200} = \frac{2421750}{11250} = 215.27\,\text{mm}$$

$$I_{xx} = \frac{1}{12} \times 150 \times 20^3 + 150 \times 20 \times 205.27^2 + \frac{1}{12} \times 470^3 \times 15$$

$$+ \frac{1}{12} \times 470 \times 15 \times 39.7 + \frac{1}{12} \times 120 \times 10^3 + 120 \times 10 \times 279.73^2$$

$$= 100000 + 126407318.7 + 8651916.67 + 927352.83 + 10000 + 93898647.48$$

$$= 229905235.7\,\text{mm}^4$$

$$y_{max} = 500 - 215.27 = 284.73\,\text{mm}.$$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{229905235.7}{284.73} = 929661.78\ \text{mm}^3$$

Area of the section is 11250 mm^2.

We know that the NA corresponding to plastic section modulus divides the cross-section into two equal areas. Let the distances of the plastic NA from the top and bottom edges be y_1 and y_2. Area of compression zone = area of tension zone

$$\frac{11250}{2} = 5625 \text{ mm}^2$$

or

$$150 \times 20 + 15 \times (y_2 - 20) = 5625$$

$$y_2 = 195 \text{ mm} \quad \text{and} \quad y_1 = 500 - 195 = 305 \text{ mm}$$

Plastic section modulus,

$$S = 120 \times 10\,(305 - 5) + 15 \times 295 \times 147.5 + 15 \times 175 \times 87.5 + 150 \times 20 \times 185$$

$$= 360000 + 652687.5 + 229687.5 + 555000 = 1797374.5 \text{ mm}^3$$

$$\text{Shape factor} = \frac{S}{Z} = \frac{1797374.5}{929661.78} = 1.93$$

23.8 LOAD FACTOR

The ratio of the collapse load to the working load or service load is called the *load factor*, i.e.,

$$\gamma_f = \frac{W_u}{W} \qquad (4)$$

where γ_f is the load factor, W_u is the collapse load or limit load, or ultimate load and W is the working load.

It represents the margin of safety with respect to the ultimate collapse load. The load factor connects the working load directly with the collapse load which is of practical significance at a much greater value as compared to the yield load. By selecting an appropriate load factor, a designer can limit the probability of collapse to an acceptable low value. Its value depends on the nature of loading, boundary conditions, and cross section of the element.

We assume here the maximum bending moment corresponding to working load W be M_{max}. Similarly, fully plastic moment corresponding to collapse load W_u be M_P. We know that bending moment at a given section is directly proportional to load. Therefore, $M \propto W$ or $M = \beta W$. For example, in the case of simply supported beam, $M_{max} = WL/4$ and hence $\beta = L/4$. Likewise, $M_P \propto W_P$ or $M_P = \beta W_u = \beta \gamma_f W$. Now, $M_P/M_{max} = \gamma_f$. We know that elastic section modulus, $Z = M_{max}/\sigma_b$ where σ_b is the allowable stress in bending. The plastic section modulus, $S = M_P/\sigma_y$

$$\therefore \quad \frac{S}{Z} = \frac{M_P}{\sigma_y} \div \frac{M_{max}}{\sigma_b} = \frac{M_P}{M_{max}} \frac{\sigma_b}{\sigma_y} \qquad \text{(a)}$$

However, $\dfrac{M_P}{M_{max}} = \gamma_f$ and $\dfrac{S}{Z} = \eta$

and $(\sigma_y/\sigma_b) = \gamma_s$ = factor of safety in elastic method.
From these quantities, we can rewrite Eq. (a) as

$$\eta = \frac{\gamma_f}{\gamma_s}$$

or $$\gamma_f = \eta \times \gamma_s \qquad (5)$$

Equation (**5**) shows that the load factor is equal to the shape factor multiplied by the factor of safety used in elastic design.

- LOAD FACTOR
- SHAPE FACTOR
- PLASTIC SECTION MODULUS
- FACTOR OF SAFETY (IN ELASTIC METHOD)

load factor i.e., $\gamma_f = \frac{W_u}{W}$

The ratio of plastic moment to the yield moment is called the *shape factor*, η

$$\therefore \quad \eta = \frac{M_P}{M_y} = \frac{\sigma_y S}{\sigma_y Z} = \frac{S}{Z}$$

The plastic section modulus, $S = M_P/\sigma_y$

$(\sigma_y/\sigma_b) = \gamma_s =$ factor of safety in elastic method.

where σ_b is the allowable stress in bending

MOMENT–CURVATURE RELATIONSHIP

We know that *curvature* is the relative rotation of two sections separated by unit distance. Let us consider sections *ab* and *cd* shown in **Fig. 1** separated by a distance *dx*. They rotate by an angle $d\theta$ as in **Fig. 1**. Then

$$d\theta = \frac{dx}{\rho} = \frac{\varepsilon dx}{y} \quad \therefore \quad \frac{1}{\rho} = \frac{\varepsilon}{y}$$

Now, we denote $\frac{1}{\rho} = \phi$

1/ρ=Ø is the curvature of the NA

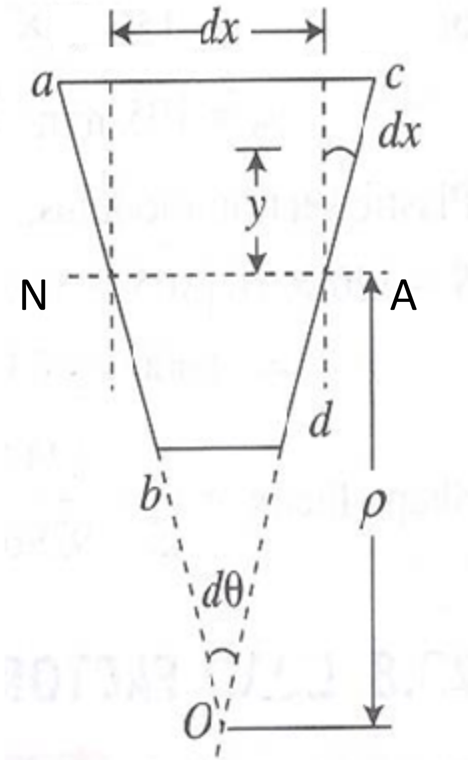
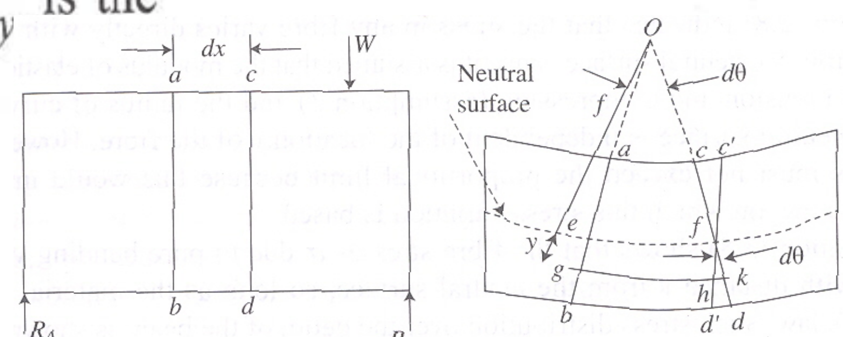


Fig. 1 Curvature.

$$\text{So, } \phi = \frac{1}{\rho} = \frac{\varepsilon}{y} = \frac{\sigma}{Ey} \qquad (6)$$

Now, let us consider a rectangular section. If ϕ_y is the curvature of the beam at the first yield, then

$$\phi_y = \frac{\varepsilon_y}{\left(\frac{d}{2}\right)}$$



(a) Simple straight beam (b) Bent beam

While the beam bends under the load, as shown in Fig. 3.39, the top fibre *ac* is shortened and the bottom fibre *bd* is lengthened. Somewhere in between the top and the bottom of the beam, there is a layer of fibres, indicated as *ef* in Fig. 3.39(b), which remain unchanged in length. This is called the *neutral surface*. The intersection of this neutral surface with the axial plane of symmetry is called the *neutral axis of the beam*. Its intersection with the plane of any cross section is called the *neutral axis of that section*. After deformation, the planes of two adjacent cross sections *ab* and *cd* intersect at *O*. The angle between these planes is denoted by $d\theta$ which is expressed as $d\theta = dx/\rho$ where $1/\rho$ is the curvature of the neutral axis of the beam.

If a line $c'd'$ is drawn through *f* parallel to *ab*, then it is clear that fibre *ac* is shortened by an amount cc' and is in compression, and that fibre *bd* is lengthened by an amount $d'd$ and is in tension. Line $c'd'$ indicates the original orientation of the cross section *cd* before bending. The deformation of a typical fibre *gh* located at a distance *y* from the neutral surface is now considered. Its elongation *hk* is the arc of a circle of radius *y* subtended by the angle $d\theta$ and is given by

$$\delta = hk = y d\theta \qquad \text{(a)}$$

The strain is found by dividing the deformation by the original length *ef* of the fibre:

$$\varepsilon = \frac{\delta}{L} = \frac{y d\theta}{ef} \qquad \text{(b)}$$

With the radius of curvature of neutral surface being ρ, as stated above, the curved length *ef* is equal to $\rho d\theta$; whence the strain becomes

$$\varepsilon = \frac{y d\theta}{\rho d\theta} = \frac{y}{\rho} \qquad (3.7)$$

If a fibre on the concave side of the neutral surface is considered, the distance *y* will be negative and the strain is also negative. Thus, all fibres on the convex side of the neutral surface are in tension while those on the concave side are in compression. Experiments indicate that the longitudinal deformation of fibres is the same as in simple tension and compression.

Assuming that the material is homogeneous and obeys Hooke's law (assumption 5), the stress in fibre *gh* is given by

$$\sigma = \varepsilon E = \left(\frac{E}{\rho}\right) y \qquad (3.8)$$

In case the beam is partially plasticized then the distribution is shown in Fig. 2

In this case, the middle layer on both sides of NA remains elastic and this layer is known as the *elastic core*. The distance of the farthest fibre which is still elastic is y_o shown in Fig. 2. Beyond this up to outer fibre the section has fully plasticized. We say the section is in elasto-plastic stage. The maximum elastic strain is ε_y. Then,

$$\frac{y_o}{d} = \frac{y_o}{\varepsilon_y}\frac{\varepsilon_y}{d} = \frac{1}{\phi}\frac{\phi_y}{2} = \frac{\phi_y}{2\phi} \qquad \text{(a)}$$

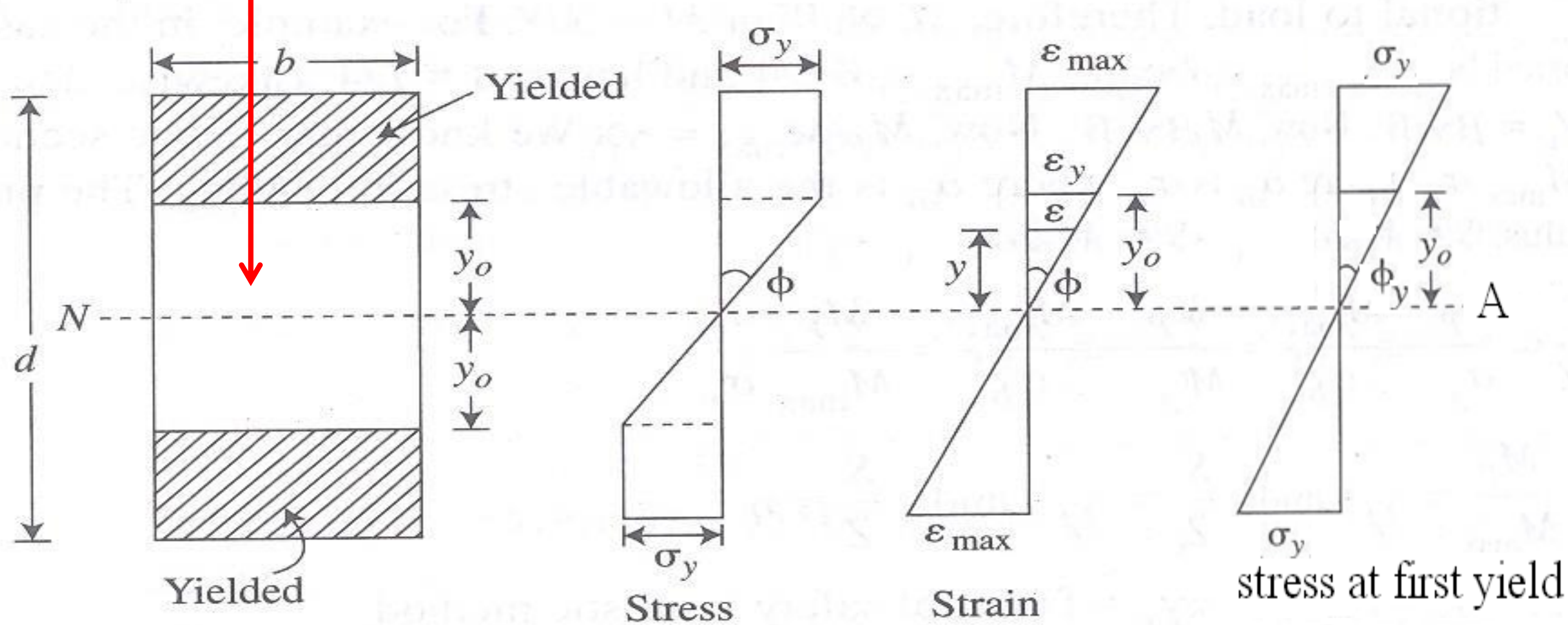


Fig. 2. Partially plasticized section.

The total moment of resistance is combination of the moment M_1, resisted by the elastic core and the moment M_2 resisted by the plastified fibres in the extreme region of the section. Therefore,

$$M_1 = \sigma_y \frac{b(2y_o)^2}{6} = \frac{2}{3}\sigma_y (by_o)^2 \qquad \text{(b)}$$

and $$M_2 = \sigma_y \left[\frac{bd^2}{4} - \frac{b}{4}(2y_o)^2\right] = \sigma_y \left[\frac{bd^2}{4} - by_o^2\right] \qquad \text{(b}'\text{)}$$

So, total moment, $M = M_1 + M_2$

$$= \frac{2}{3}\sigma_y by_o^2 + \sigma_y \left[\frac{bd^2}{4} - by_o^2\right] \qquad \text{(c)}$$

$$= \sigma_y \frac{bd^2}{4} + \sigma_y \left[\frac{2}{3}by_o^2 - by_o^2\right] = \sigma_y \frac{bd^2}{4} - \sigma_y \frac{by_o^2}{3}$$

$$= \sigma_y \frac{bd^2}{6}\left[\frac{3}{2} - \frac{2y_o^2}{d^2}\right] \qquad \text{(d)}$$

We know $M_y = \sigma_y bd^2/6$. Substituting in Eq. (d), we get

$$M = M_y \left[\frac{3}{2} - \frac{2y_0^2}{d^2}\right] \qquad (\mathbf{6})$$

Substituting Eq. (a) in Eq. (**6**), we get

$$M = M_y \left[\frac{3}{2} - 2\left(\frac{\phi_y}{2\phi}\right)^2\right]$$

$$\therefore \quad \frac{M}{M_y} = \frac{3}{2}\left[1 - \frac{1}{3}\left(\frac{\phi_y}{\phi}\right)^2\right] \qquad (\mathbf{7})$$

Equation (7) gives the moment curvature relationship in the elasto-plastic stage for rectangular section.

The moment curvature relation in the elastic stage is given by

$$\frac{M}{M_y} = \frac{\varepsilon}{\varepsilon_y} = \frac{\phi}{\phi_y} \qquad (8)$$

We can show the moment curvature relationship for a rectangular section as per Eqs (7) and (8) in Fig. 3

It can be observed from Fig. 3 that the $M-\phi$ relationship is linear in the elastic range and curvilinear in the plastic range. The $M-\phi$ curve becomes

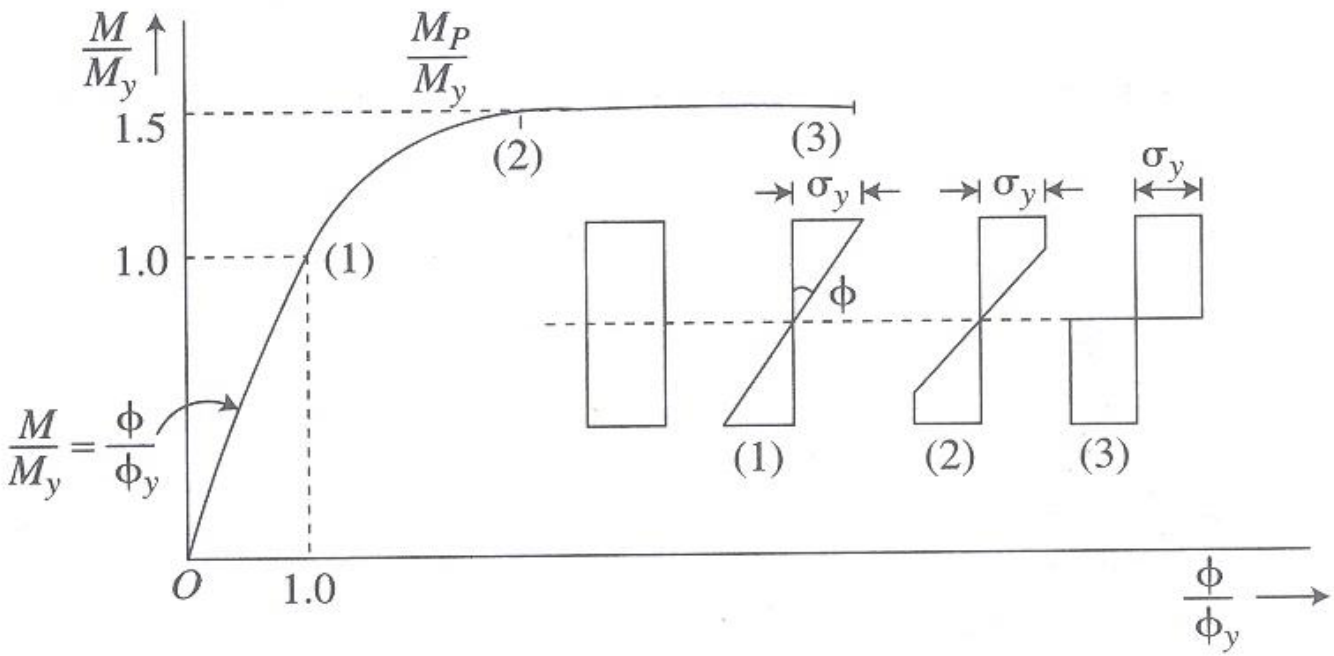


Fig. 3 Non-dimensional $M-\phi$ curve for rectangular section.

asymptotic to the horizontal line shown dotted corresponding to $M/M_y = 1.5$, i.e., when M reaches the value of M_P.

The *moment–curvature relationship* is an important aspect of plastic analysis. In an unloaded beam the curvature is zero. As we increase the load on the beam correspondingly the moment increases as a result of which its curvature also increases linearly up to the point (1) in Fig. 3. From point 0 to point (1), it is called *elastic range*. With the yielding of fibre in the section at yield moment M_y, the linear relationship ceases. With further increase in moment, the curvature increases at a faster rate indicating that the yield spreads into other fibres inside the depth of the section. As the moment attains the fully plastic value, curvature tends to infinity. This indicates that the section is fully plasticized. When at a particular section along the length of the beam, moment reaches the value of M_P, whereas the value of the moment at other sections on either side of it still remains lower than M_P. At a fully plasticized section, the curvature becomes infinitely large. Therefore, a finite change of slope can occur over an infinitely small length of the member at this plastified section. So, the member will rotate about this plastified section as though a hinge has been inserted there.

EFFECT OF AXIAL LOAD ON PLASTIC MOMENT

Consider a rectangular beam whose cross-section is shown in Figure (a) below. The beam is under the action of combined effect of axial thrust and the bending moment. Let the section be fully plastic under this combined effect of axial force P and a bending moment M_p'

In the figure, the areas yielding in compression are shaded while the areas under tension are hollow. The gross cross-section may be considered as made up of areas as shown in Figure (b) & (c). The resultant of the stresses acting on the area in Figure (b) is equal to the applied axial load .

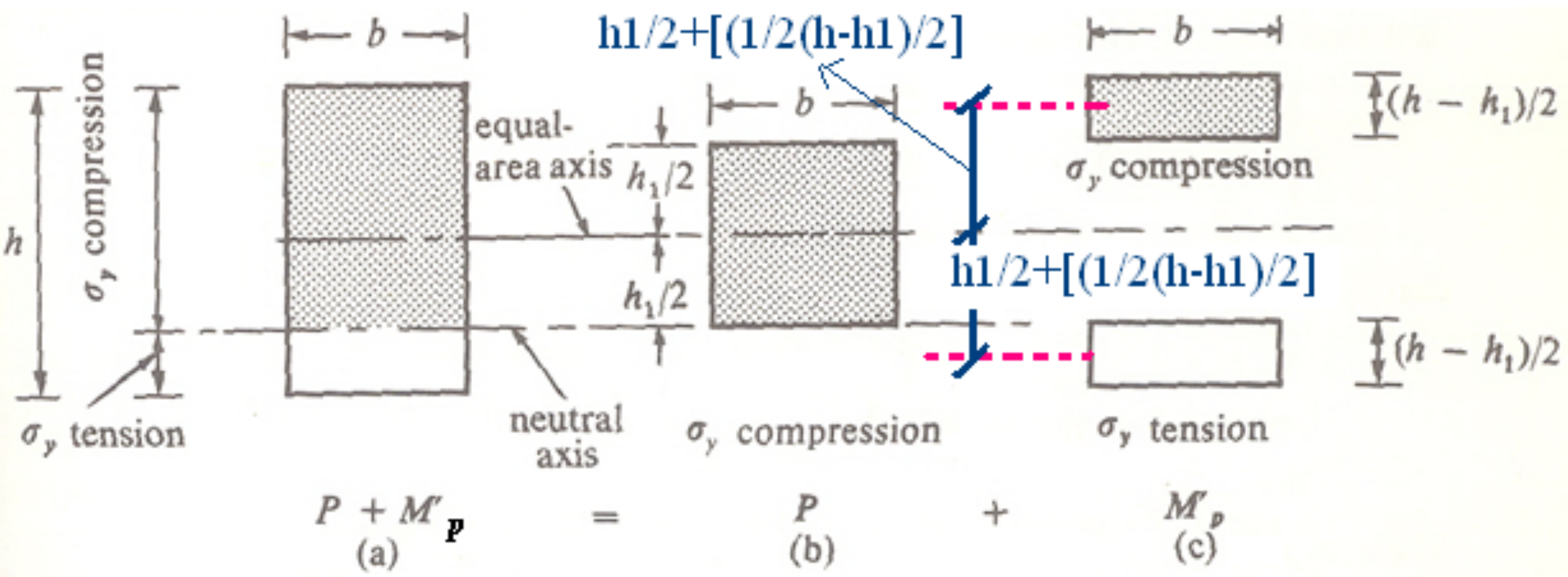


Fig. [(a) = (b) + (c)]

In the figure, the areas yielding in compression are shaded while the areas under tension are hollow. The gross cross-section $b \times h$ may be considered as made up of areas as shown in Figure (b) & (c). The resultant of the stresses acting on the area in Figure (b) is equal to the applied axial load P.

$$P = \sigma_y b h_1$$
$$= \frac{h_1}{h} \sigma_y bh$$
$$= nP_o \quad (1)$$

Where $P_o = \sigma_y bh$ is called the **squash load** of the section. We know the ratio $\frac{P}{P_o}$ is called the **squash load ratio** and is usually denoted by ***n.*** For a rectangular section, $n = \frac{h_1}{h}$, as shown by Equation (1).

For the beam shown in Figure (a) above, under the **combined action** of axial force P acting through the Plastic NA of the section of Figure (b) and an applied bending moment M_p', the shaded section shown in Fig (b) shall be under pure compression, the and hence cause no **FLXURE.**

The resultant of the stresses acting on the areas as shown in Fig (c) shall only be contributed by Fig (c), is a couple equal to the applied plastic moment M_p'. From the Figure (a), (b) & (c) above, we can write

$$M_p' = 2 \times \{\sigma_y \times b \times [(h - h_1)/2]\} \times \left\{(h_1/2) + \frac{1}{2} \times ((h - h_1)/2)\right\}$$

On simplifying, we get

$$M_p' = \sigma_y \times \frac{bh^2}{4} - \sigma_y \times \frac{bh_1^2}{4}$$

$$or \quad M_p' = M_p - n^2 M_p \qquad (2)$$

Where M_p' is the plastic moment of the section in the presence of axial load ***P***, and M_p is the plastic moment in pure bending. Dividing Eqn (2) by σ_y.

$$Z_p' = Z_p - n^2 Z_p \qquad (3)$$

Where Z_p' plastic section modulus in the presence of axial &

Z_p that in pure bending. Eqn. (2) shows that the plastic moment of a rectangular section is always reduced by the presence of axial load acting in the plane of equal area axis. It should also be noted that Eqn. (1) is true irrespective of whether the load is compressive or tensile.

For a mono-symmetrical section, such as a T section, we have to define carefully the term axial load. If an axial load is defined as one acting in the plane of the equal-area axis, it is clear from the analysis in Fig. below that an axial load will always reduce the plastic moment, by an amount equal to $\sigma_y \times [Z_p$ of area in Fig. (b)].

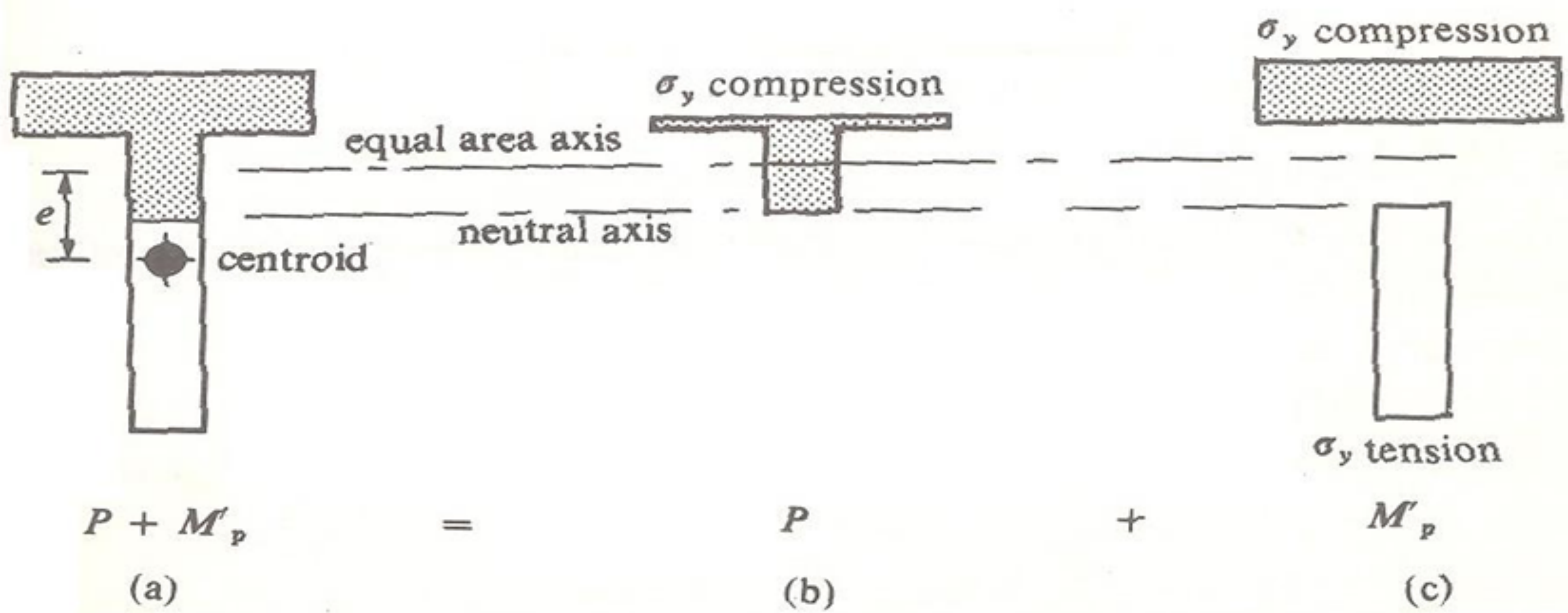
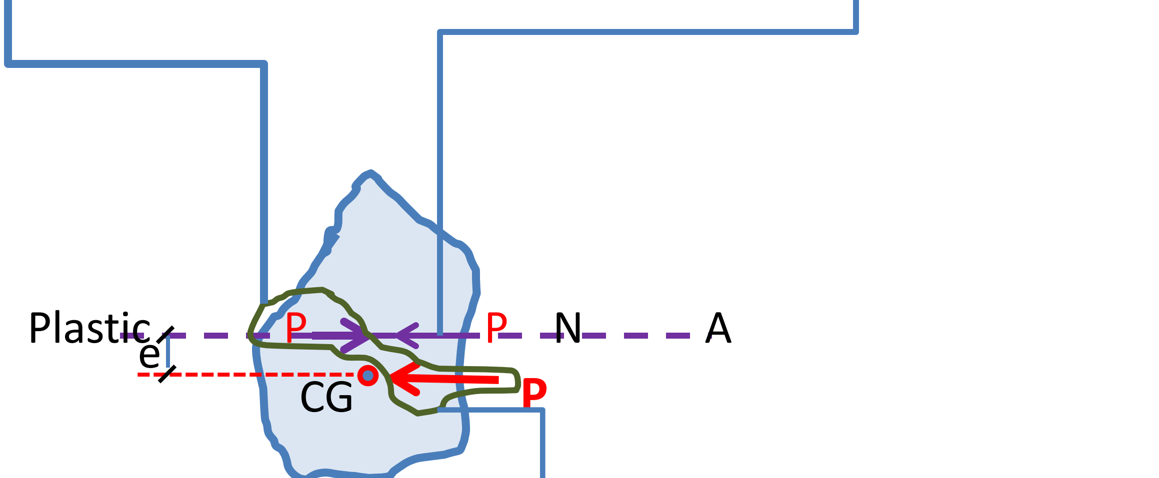


Fig. [(a) = (b) + (c)]

If an axial load is defined as one acting through the centroid of the section, then it is effectively equal to a load acting in the plane of the equal-area axis plus an additional bending moment *Pe*, where *e* is the eccentricity of the centroid from the equal-area axis. If the sign of this additional moment is favourable then the plastic moment may appear to increase; this increase is, of course, purely illusory.



Collapse Load and Collapse Mechanism

The load Carrying capacity of any frame or beam depends only on the value of Plastic Moment of Resistance M_P.

In the plastic analysis, it is assumed the elastic deformations are small and that the behaviour is Perfectly Plastic or Rigid-Plastic.

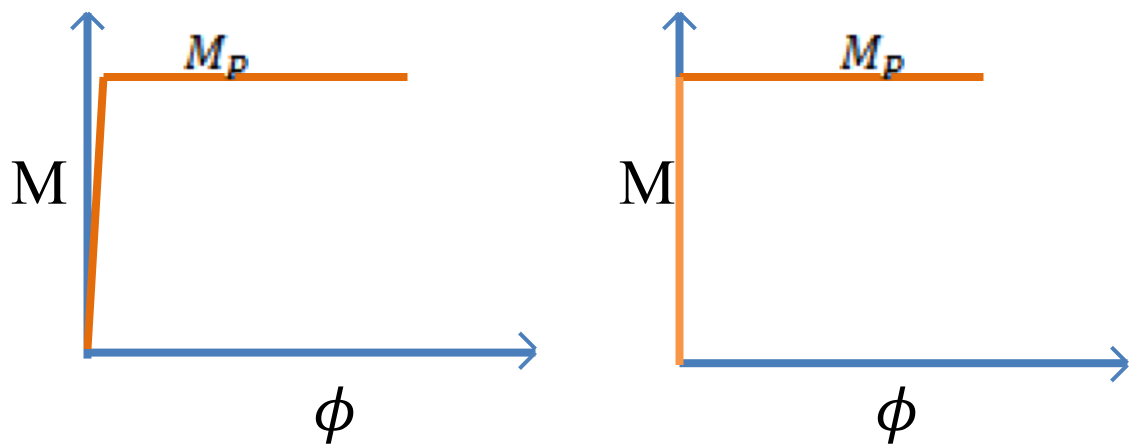


Fig. 14.6-1

(a) Elastic-plastic (b)Rigid-plastic

In all further discussions, unless otherwise specified, we shall assume a rigid-plastic moment curvature relationship. It is also assumed that the effect of axial and shear deformation are very small and hence considered negligible.

EXAMPLE: To determine the collapse load

As a first example of the determination of the collapse load of a structure by plastic analysis, consider the two-span beam in Fig. 14.6-2(a), having a uniform cross section of plastic moment of resistance M_p. The beam supports a **working load *P***, which is sufficiently low *for stresses everywhere to be within the elastic range*. The bending moment diagram is as in Fig. 14.6-2(b). As the load *P* is progressively increased to say ***βP***, the bending moment at section *D* reaches ***M***M_p and a plastic hinge forms there, as shown in Fig. 14.6-2(c). The beam is originally statically indeterminate and has one redundancy, but the formation of a plastic hinge removes that redundancy so that the beam in Fig. 14.6-2(c) is statically determinate. The bending moment diagram is now represented by *abdc* in Fig. 14.6-2(d). As the load is further increased, the **bending moment at D remains constant** at the full plastic value M_p, while that at B continues to grow until eventually, at a load of say ***λP***, the moment at B also reaches M_p.

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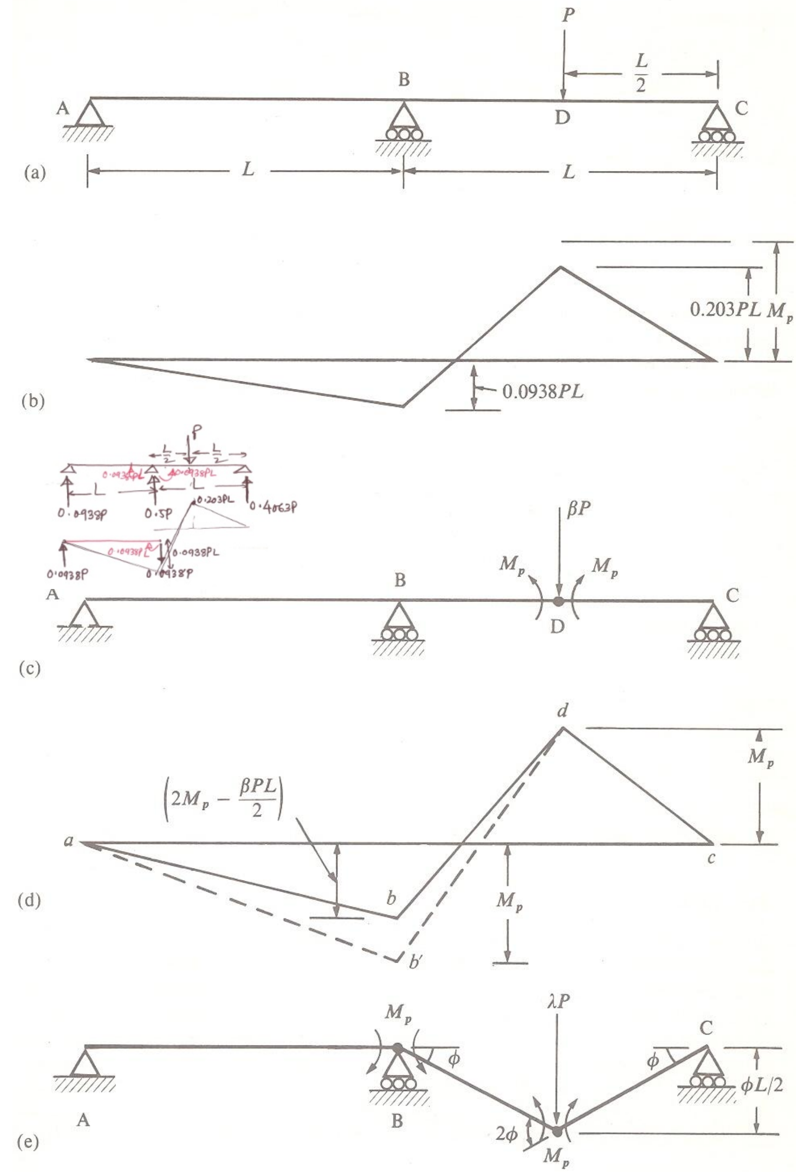


Fig. 14·6–2

The bending moment diagram is now ab'dc in Fig. 14.6-2(d), and there are two plastic hinges - at B and at D. Reference to the moment-curvature curve in Fig. 14.6-I(b) shows that the beam now undergoes unrestrained rotation at B and D; in other words, the structure has become a **mechanism**. The load **λP** at which the structure collapses as a **mechanism** is called the **collapse load** or the **ultimate load** and the factor **λ** which is the ratio of the **collapse load** to the **working load**, is called the **collapse load factor**, or simply the **load factor**. The collapse load, and hence the load factor, can easily be determined from Fig. 14.6-2(e). Consider a plastic hinge rotation ϕ at B; then from the geometry of Fig. 14.6-2(e), that at hinge D must be 2ϕ.

Therefore

Work done by load $= \lambda P(\phi L/2)$

Work dissipated in the hinges $= M_p\phi + M_p(2\phi)$

$= 3M_p\phi$

Hence the work **equation** (sometimes called the **collapse equation**) is

(Using principle of Virtual work)

$$\lambda P\phi L/2 = 3M_p\phi \tag{14.6–1}$$

therefore $\lambda P = 6M_p/L$

$$\lambda = 6M_p/PL$$

This method of solution is called **work method** or **mechanism method** wherein the ***collapse mechanism is identified and the collapse load is then obtained from the work eqn***.

METHOD OF PLASTIC ANALYSIS

(1) Work Method or Mechanism Method or Virtula Work Method
(2) Statical Method or Graphical Method (Lower Bound Theorem)
(3) Uniqueness Theorem
(4) Kinematic Method (Upper Bound Theorem)

(1) **Work Method or Mechanism Method** : Already discussed

(2) **Statical Method**:
In this method, ***the redundant moments are selected and the bending moment diagram is constructed by superimposing the free moment-diagram on to the redundant-moment diagram in such a way that the mechanism is formed — the value of the collapse load is then calculated from statics.***

Example: Consider again the beam of Fig. 14.6-2 (a)

(2) **Statical Method**: or Lower Bound Theorem.

It states that ***'if a distribution of bending moments can be found such that the structure is in equilibrium under the external loading and such that nowhere is the plastic moment of resistance M_p exceeded, then the structure will not collapse under that loading***-however 'unlikely' that distribution of moments may appear. The theorem is often referred to as the safe theorem. (The proof of this theorem will be given in Section 14.9).

In this method, ***the redundant moments are selected and the bending moment diagram is constructed by superimposing the free moment-diagram on to the redundant-moment diagram in such a way that the mechanism is formed the value of the collapse load is then calculated from statics.***

Example: Consider again the beam of Fig. 14.6-2 (a)

Example:

As an example, consider again the beam in Fig. 14.6-2(a)- the collapse mechanism of Fig. 14.6-2(e) is redrawn in Fig. 14.6-3(a). *Suppose the continuity moment at the intermediate support B is selected as the redundant*, then the bending moment diagram may be sketched as in Fig. 14.6-3(b), in which ***ab_2c is the redundant-moment diagram***, and **b_1d_2c is the free moment diagram due to the load λP** acting on the simple span BC. If the magnitude of b_1b_2 is so chosen that $b_1b_2 = d_1d_2 = M_p$, then the beam will collapse in the mechanism of Fig. 14.6-3(a). From the geometry of Fig. 14.6-3(b):

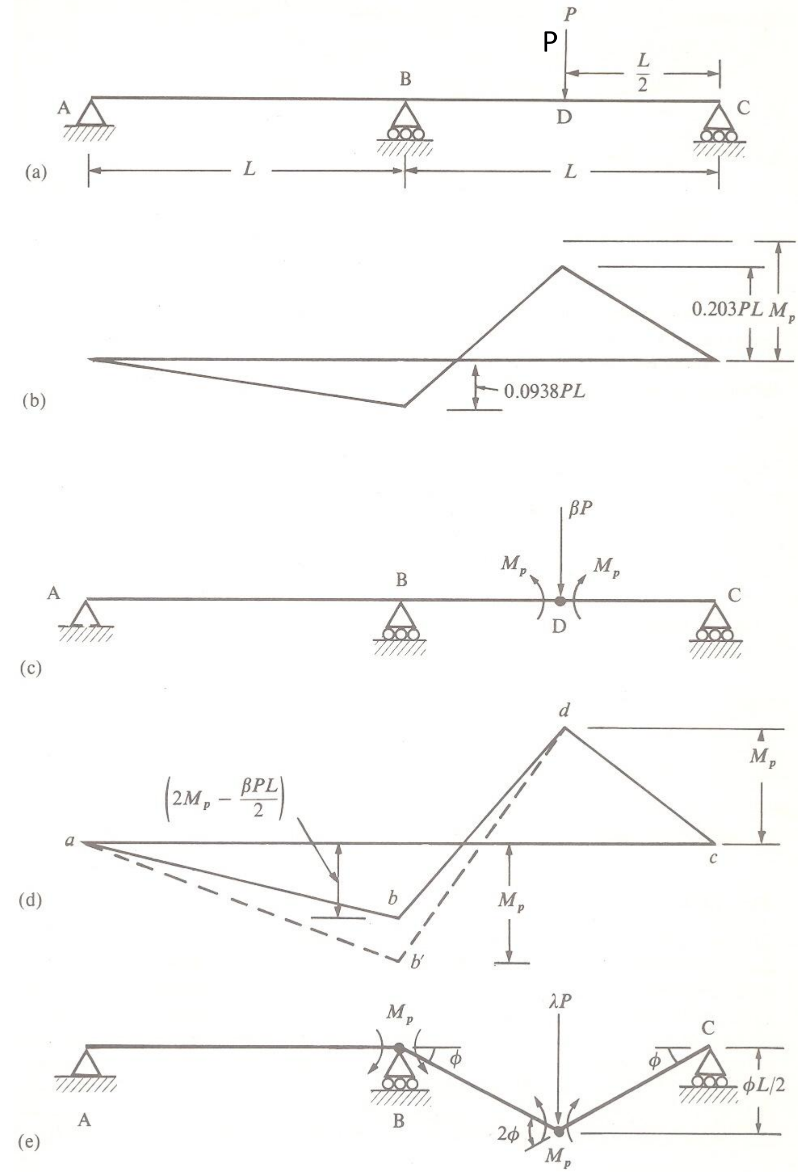


Fig. 14·6–2

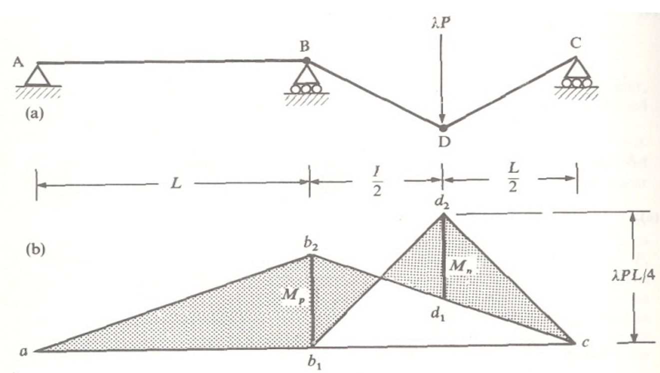


Fig. 14·6–3

$$d_1d_2 = \lambda PL/4 - b_1b_2/2$$

That is $\quad M_p = \lambda PL/4 - M_p/2$

Therefore $\quad \lambda P = 6M_p/L \quad$ agreeing with Eqn 14.6–1.

Necessary conditions for determination of collapse load

The following conditions are must to be satisfied to have collapse of the structure.

(1) the ***Mechanism Condition***
(2) the ***equilibrium condition*** &
(3) the ***yield condition***

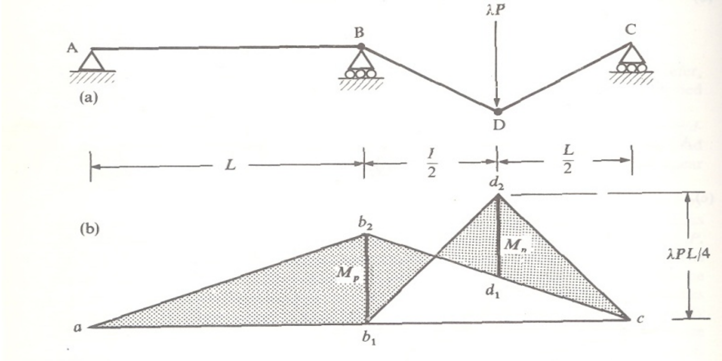


Fig. 14·6–3

With reference to the Fig. 14.6-3,

The ***Mechanism Condition*** is satisfied *i.e.* sufficient (*in this case=2*) no. of plastic hinges have formed to convert the structure into mechanism.

The ***equilibrium condition*** is satisfied (ref. Fig 14.6-3(b)) where the bending moment distribution is in equilibrium with λP (*meaning that the BM induced by λP satisfy* ΣM=0 *at the locations of Plastic hinges* (*i.e.* at points B and D)).

The ***yield condition*** is meant the condition that the bending moment (ref. Fig 14.6-3(b)) nowhere exceeds Plastic moment of resistance M_P. (***implies nowhere, the ordinate of BMD(shaded) exceeds*** M_p).

These three conditions are necessary and sufficient for the determination of the collapse load factor of a structure.

(3) Uniqueness Theorem: ***It sates that if a bending moment distribution can be found which satisfies the three conditions of Mechanism, equilibrium and yield --- then the load which correspond to such a moment distribution will be true collapse load.***

Example 14.6–1. The propped uniform cantilever in Fig. 14.6–4(a) is of plastic moment M_p. Determine the value of P at collapse using (a) the work method (b) the statical method.

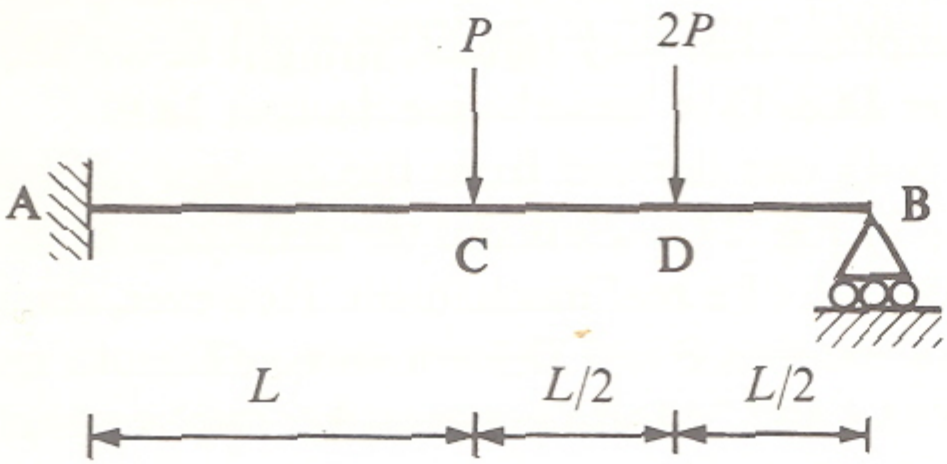


Fig. 14·6–4(a)

Solution:

(a) Using The Work Method (or the Virtual Work Method)

There are three possible collapse Mechanism as shown in Figs. 14.6 (b), (c) and (d).

Finding out the **collapse load** for each of these Mechanisms as shown below.

There are three critical sections where BMs could be maximum *viz*. Fixed support, below point load P and below point load 2P. Further, we know that I=1 and hence 02 hinges are sufficient to convert it into MECHANISM. Now examining all the three possible mechanisms. It is observed that the one in Fig. 14.6-4(b) gives the lowest collapse load. This means that as the magnitude of *P* is gradually increased from zero, the collapse mechanism in Fig. 14.6-4(b) will be the first to form (correspond to lowest load), when *P* reaches $5M_p/4L$.

The other two mechanisms cannot form unless this one is prevented from forming, for example by strengthening the cross sections at points where hinges would have formed. We therefore conclude that $P = 5M_p/4L$ is the correct collapse value as the first mechanism at the lowest load will cause the collapse of the structure and the other two mechanism will not be forming.

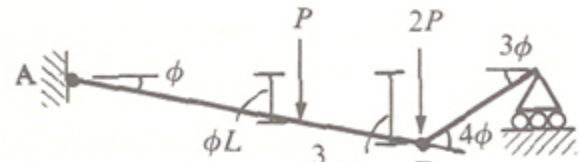


Fig. 14·6–4(b)

The work equation is

$$P(\phi L) + 2P(3\phi L/2) = M_p\phi + M_p(4\phi)$$

Therefore $P = 5M_p/4L$ $= 1.25 \frac{M_p}{L}$

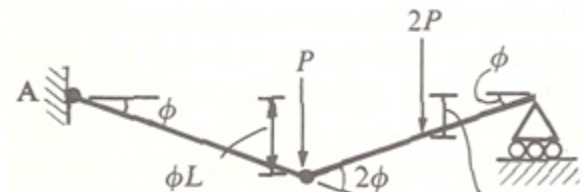


Fig. 14·6–4 (c)

$$P(\phi L) + 2P(\phi L/2) = M_p\phi + M_p(2\phi)$$

Therefore $P = 3M_p/2L$ $= 1.5 \frac{M_p}{L}$

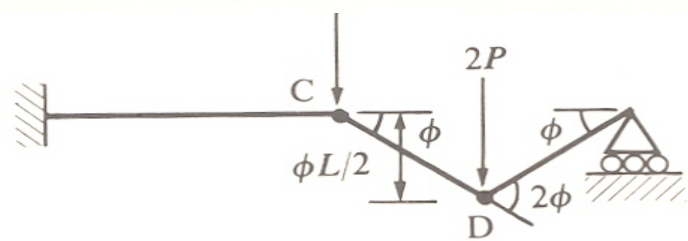
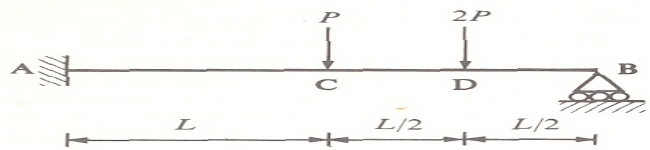


Fig. 14·6–4(d)

$$2P(\phi L/2) = M_p\phi + M_p(2\phi)$$

Therefore $P = 3M_p/L$



(b) Using the Statical Method:

Figs. 14.6-4(e), (f) and (g) show three bending moment diagrams. Each diagram has been so drawn that the moment ordinate is exactly equal to the plastic moment M_p at two sections. In Figs. (e) and (f), $a_1 a_2 b$ is the moment diagram due to the **redundant moment M_p at A**, and $a_1c_2d_2b$ is the simple-span moment diagram due to the external loads P and $2P$.

In Fig. (g) the moment **M_p at section C has been selected as the redundant**; that is, the triangle $a_1a_2c_2bc_1a_1$ is the redundant moment diagram (The reader should verify this. Hint: the moment *Mp* at C produces shear forces.), and $a_la_3c_ld_2b$ is the moment diagram for the loads P and $2P$ acting on the beam with a hinge at C. The collapse values of P can be calculated from the geometry of the three bending moment diagrams.

The three values for P obtained by the statical method agree with those obtained by the work method. As before, we conclude that the lowest value, namely $P = SM_p/4L$, is the correct one.

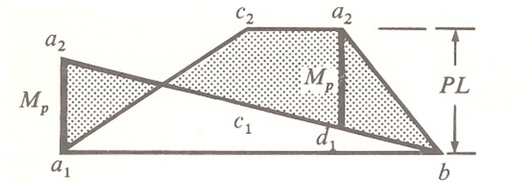


Fig. 14·6–4(e)

$$d_1d_2 = PL - a_1a_2/4$$

Therefore $M_p = PL - M_p/4$

or $P = 5M_p/4L$

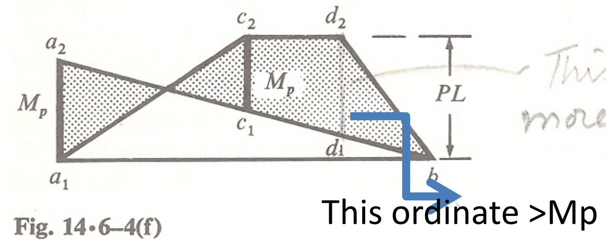


Fig. 14·6–4(f)

$$c_1c_2 = PL - a_1a_2/2$$

$$M_p = PL - M_p/2$$

Therefore $P = 3M_p/2L$

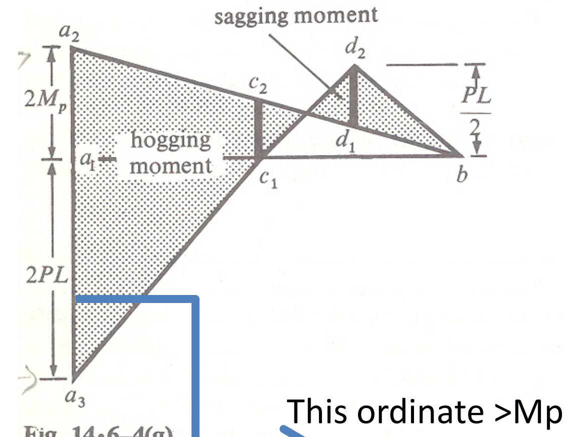
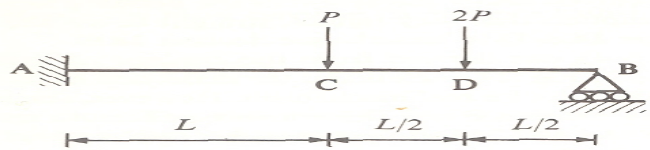


Fig. 14·6–4(g)

$$d_1d_2 = PL/2 - c_1c_2/2$$

$$M_p = PL/2 - M_p/2$$

Therefore $P = 3M_p/L$



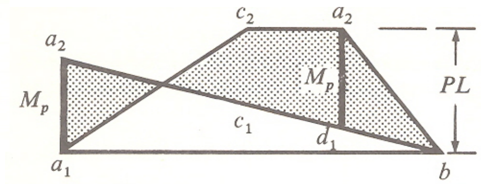


Fig. 14·6–4(e)

COMMENTS

In the statical method above, we need not have calculated the collapse value of *P* for all the three bending moment diagrams. A closer examination will immediately reveal that both Figs. 14.6-4(f) and (g) violate the yield condition. In Fig. 14.6-4(f), the moment ordinate d_1d_2 exceeds M_p, which means that the collapse mechanism in Fig. 14.6-4(c) cannot occur unless plastic hinge formation is prevented at D by strengthening the cross section there. Similarly, in Fig. 14.6-4(g), $a_2\,a_3$ exceeds M_p; again, the mechanism in Fig. 14.6-4(d) cannot occur unless plastic hinge formation at A is deliberately prevented. The bending moment diagram in Fig. 14.6-4(e), on the other hand, satisfies the three conditions of mechanism (with plastic hinges at A and D), equilibrium (by the manner of its construction) and yield (since moment ordinates nowhere exceed M_p). Therefore we can at once conclude from the **uniqueness theorem** that the corresponding collapse load is the correct one; there is in fact no need to consider any other mechanisms.

$d_1d_2 = PL - a_1a_2/4$

Therefore $M_p = PL - M_p/4$

or $P = 5M_p/4L$

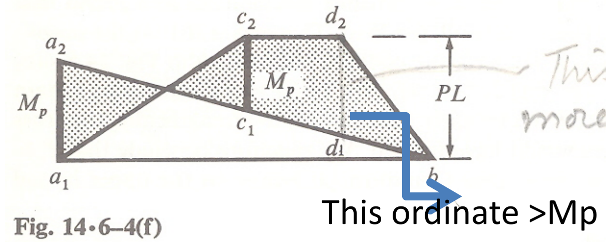


Fig. 14·6–4(f)

$c_1c_2 = PL - a_1a_2/2$

$M_p = PL - M_p/2$

Therefore $P = 3M_p/2L$

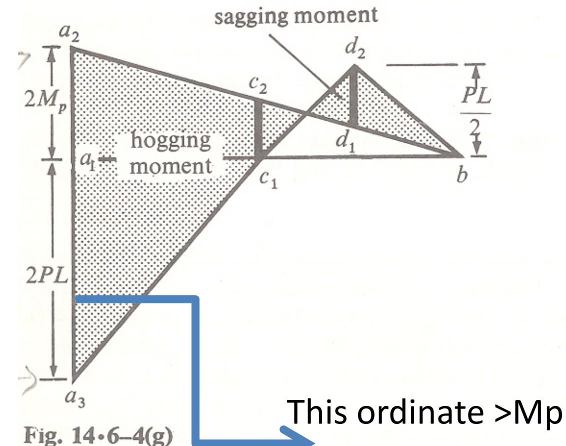


Fig. 14·6–4(g)

$d_1d_2 = PL/2 - c_1c_2/2$

$M_p = PL/2 - M_p/2$

Therefore $P = 3M_p/L$

(4) Upper Bound Theorem: (Also called as Kinematic Theorm)

It states that 'for a given structure subjected to a given loading, the magnitude of the loading which is found to correspond to any assumed collapse mechanism must be either greater than or equal to, but cannot be less than, the true collapse load'.

Therefore, in an analysis we simply compute the collapse load for each possible mechanism and **accept the lowest value** as the correct one, as we did in the work method above.

In this method, number of possible mechanisms are studied. A statical check is applied by ensuring that the moment ordinate nowhere exceeds the plastic moment M_p for each mechanism correspond to collapse bending Moment Diagram.

If the moment ordinate nowhere exceeds the plastic moment M_p then the **uniqueness theorem** guarantees that this mechanism will give the true collapse load.

If M_p is exceeded somewhere, then the yield condition is not satisfied, and the search for a correct collapse mechanism must continue.

The upper bound theorem is often referred to as the **unsafe theorem**, because, interpreted in a design sense, it states that the value of the plastic moment M_p obtained on the basis of an arbitrarily assumed collapse mechanism is smaller than, or at best equal to, that actually required.

Example 14.6-2. A propped uniform cantilever is to be designed to support the loads in Fig. 14.6-6(a). Explain how the lower bound theorem may be used to select a value of the plastic moment of resistance, *Mp,* which will guarantee that the beam will not collapse under the loading.

SOLUTION Fig. 14.6-6(b), (c) and (d) show three bending moment diagrams which are in equilibrium with the external loads *P* and *2P.* These are discussed in turn:
Fig. 14.6-6(1)): The bending moment diagram is obtained by superposition of the simple-beam moment diagram $a_1c_2d_2b$ on to the diagram *a1a2b* due to the redundant moment *M* at the fixed end A. The value of *M* actually acting is, of course, not known,

Example 14.6-2. A propped uniform cantilever is to be designed to support the loads in Fig. 14.6-6(a). Explain how the lower bound theorem (Static Theorem) may be used to select a value of the plastic moment of resistance, *Mp,* which will guarantee that the beam will not collapse under the loading.

SOLUTION Fig. 14.6-6(b), (c) and (d) show three bending moment diagrams which are in equilibrium with the external loads *P* and *2P.*

These are discussed in turn:

Fig. 14.6-6(b)): The bending moment diagram is obtained by superposition of the simple-beam moment diagram $a_1c_2d_2b$ on to the diagram a_1a_2b due to the **redundant moment *M* at the fixed end A**. The value of *M* actually acting is, not known, but this presents no difficulties because the designer is at liberty to choose any value he considers appropriate. Suppose, he sets *M* at a value represented to scale by the ordinate a_1a_2. The largest moment ordinate is then d_1d_2. Thus, provided the plastic moment of resistance M_p exceeds d_1d_2 (= $PL - M/4$), the yield condition is

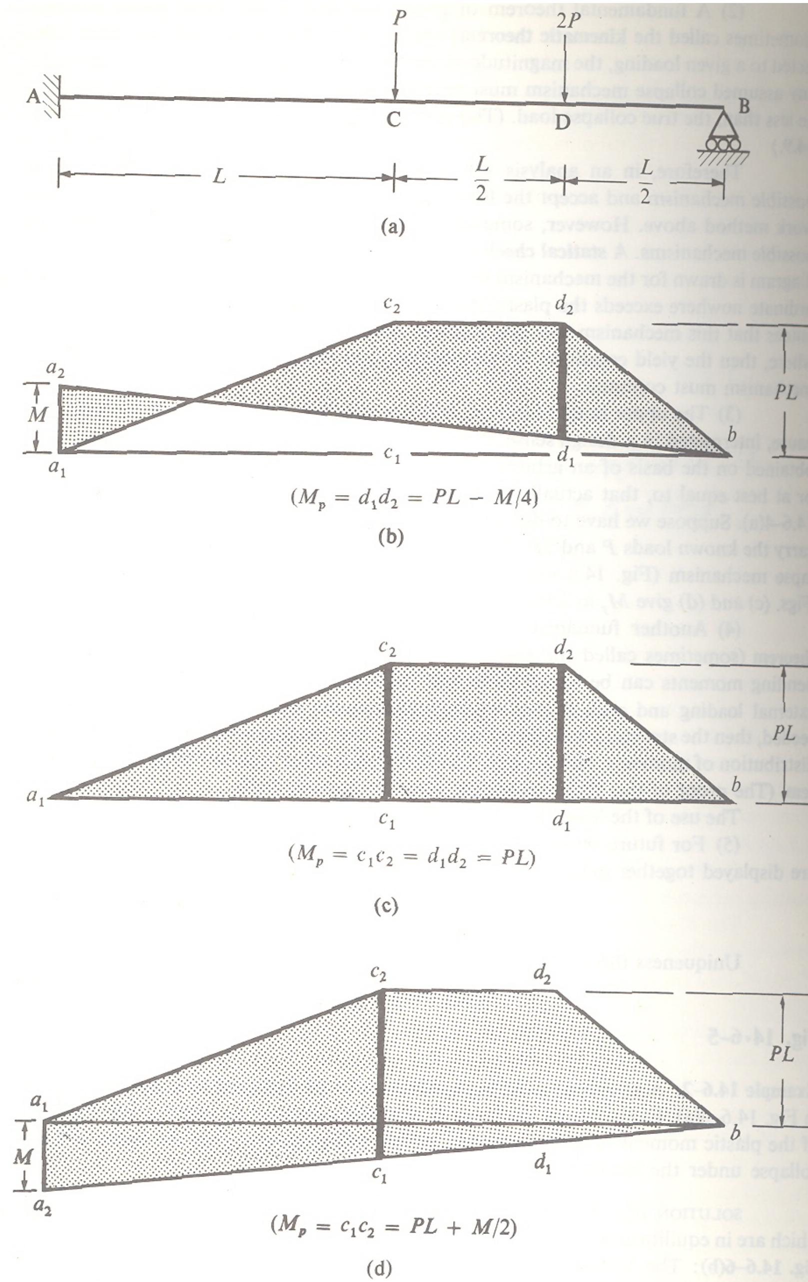


Fig. 14·6–6