COSMOLOGICAL AND THEORETICAL APPLICATIONS OF EXACT SOLUTIONS OF EINSTEIN'S EQUATIONS

OVERVIEW

The current era of Precision Cosmology has produced a large amount of high quality observational data at all astrophysical and cosmic scales whose theoretical interpretation requires a robust modelling of self-gravitating systems. Because of the high non-linear complexity of General Relativity, most research in Cosmology is undertaken either through linear perturbations on a LCDM background, or by Newtonian methods (perturbative and non--perturbative, as well as numerical simulations). While the exact solutions of Einstein's equations that are used in cosmological research, are very idealised, they are still extremely useful, as they allow researchers to probe relativistic and non-linear effects in self-gravitating systems through a mathematically tractable approach. For example: spherically symmetric Lemaitre-Tolman-Bondi (LTB) dust models have been employed (the „spherical collapse model") to predict cold dark matter virialisation and collapsing times in structure formation, as well as large scale observations fitting, while Szekeres dust models (their best known non-spherical generalisation) can be used to describe these effects for several evolving cold dark matter structures. Other classes of useful solutions are shear-free solutions (including the Stephani models) and the homogeneous but anisotropic Kantowski-Sachs (KS) and Bianchi models. Basic features of numerous observational and theoretical issues, such as observations fitting, averaging, back-reaction, gravitational entropy, cosmic censorship, can be understood (at least at a toy model level) through these exact solutions. The predictions from these models can provide valuable insight on the behavior of more realistic configurations to be studied by means of numerical methods. Therefore, understanding the properties and possible applications of the above mentioned exact solutions of Einstein's equations is an essential ingredient in the theoretical arsenal of all researchers in Astrophysics, General Relativity and Cosmology.

OBJECTIVES

The primary objectives of the course are as follows:

i) Exposing participants to the fundamentals of General Relativity Theory, Cosmology, current theoretical and observational open problems and the methodology and models used to address them
ii) Derivation, classification and theoretical properties of exact solutions of Einstein's equations that are favored for various applications in General Relativity and Cosmology
iii) Applications exact solutions in structure formation, cosmic structure modeling and fitting cosmological observations. Importance of relativistic corrections. Relation to linear perturbation theory. Numerical methods based on these models.
iv) Applications of exact solutions to address open theoretical problems in General Relativity: averaging, back-reaction, gravitational entropy, etc. Numerical methods

Modules	**A: Duration:** **February 12-23, 2016 (24 Hours)** **B: Venue** **Centre for Theoretical Physics, Jamia Millia Islamia (A Central University), New Delhi – 25, India** ***Number of participants for the course will be limited to 50 fifty.***
Who Can Attend	§ **Researchers in General Relativity and Astrophysics** § **Graduate students (Ph.D./M.Phil/M.Sc.) with basic understanding of General Relativity Theory**
Fees	The participation fees for taking the course is as follows: **Participants from abroad : US $500** **Industry/ Research Organizations: INR 10000** **Academic Institutions:** - **Faculty members: Rs. 3000/-** - **Students: Rs 1500/-** ***The above fee include all instructional materials, tutorials and assignments, laboratory equipment usage charges, 24 hour free internet facility. The participants will be provided accommodation on payment basis, subject to the availability.***

LECTURE-WISE COURSE PLAN: (FEBRUARY 12-23, 2016)

MODULE A: BASIC THEORETICAL GROUND WORK (2 DAYS)

First Day – Friday February 12, 2016

- Lecture 1 : One hour – 9.15 am – 10.15 am
  - Einstein's equations, Symmetries, Basic exact solutions: de Sitter and FLRW models, spherically symmetric static solutions. Overview of matter-energy sources.

- Lecture 2: One hour – 10.30 am – 11.30 am
  - Overview of LTB and Szekeres models in their standard representation. Shear-free solutions. KS and Biachi models. Summary of useful exact solutions.

- Tutorial 1. Two hours – 11.30 am – 1.30 pm
  - Techniques for solving Einstein's equations through a Computer Algebra System (CAS). Review of numerical methods to handle exact solutions. Demonstrations and on terminal excercises.

Second Day - Saturday February 13, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
  - Covariant curvature and kinematic objects. Re-casting Einstein's equations as a first order system of evolution equations of these covariant objects: the 1+3 fluid flow formalism.

- Lecture 2: One hour - 10.30 am – 11.30 am
  - Exact solutions as dynamical systems: expansion normalised variables, self similarity and other useful techniques.

- Tutorial 1. Two hours - 11.30 am – 1.30 pm
  - Setting up, solving and analysing simple dynamical systems based on FLRW models, non-tilted Bianchi models, self-similar LTB and Szekeres models. Demonstrations aided by CAS.

MODULE B: QUASI-LOCAL SCALAR VARIABLES (2 DAYS)

First Day – SundayFebruary 14, 2016

- Lecture 1 : One hour - 9.15 am – 10.15 am
  - Quasi-local variables and quasi-local averages in LTB models. Advantages over traditional variables.

- Lecture 2: One hour - 10.30 am – 11.30 am
  - Quasi-local variables and quasi-local averages in Szekeres models.

- Tutorial 1. Two hours - 11.30 am – 1.30 pm
  - Dynamical Systems approach to the evolution of LTB and Szekeres models in terms of quasi-local variables. Demonstrations and `on terminal` excercises aided bya CAS.

Second Day - MondayFebruary 15, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Covariant interpretation. Connection with Buchert's averaging formalism.

- Lecture 2: One hour - 10.30 am – 11.30 am
    - Quasi-local variables and quasi-local averages in more general spacetimes.

- Tutorial 1. Two hours - 11.30 am – 1.30 pm
    - Overview of theoretical applications. Case study: gravitational entropy.

MODULE C: QUASI-LOCAL VARIABLES AND COSMOLOGICAL PERTURBATIONS IN LTB MODELS. (2 DAYS)

First Day – TuesdayFebruary 16, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Quasi-local variables as `exact' perturbations. Local and asymptotic FLRW background.

- Lecture 2: One hour - 10.30 am – 11.30 am
    - Connection with standard gauge invariant cosmological perturbations. Connection with covariant fluid flow perturbations

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Growing and decaying modes as `exact' quantities. Dynamical systems in terms of these modes. Initial conditions

Second Day - WednesdayFebruary 17, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Cold dark matter structures described with LTB models in terms of quasi-local variables.
- Lecture 2: One hour - 10.30 am – 11.30 am
    - Initial conditions, growth and suppression of perturbations

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Problem solving session (demonstration and `on terminal`) with examples worked out with the aid of a CAS

MODULE D: QUASI-LOCAL VARIABLES IN SZEKERES MODELS. (2 DAYS)

First Day – ThursdayFebruary 18, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Dynamics through exact non-spherical perturbations by means of quasi-local scalars.

- Lecture 2: One hour - 10.30 am – 11.30 am
    - Exact growing and decaying modes. Connection with the Goode and Wainwright

variables

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Dynamical systems and numerical methods. Initial conditions

Second Day - FridayFebruary 19, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Modelling simple non-spherical structures. Axial symmetry
- Lecture 2: One hour - 10.30 am – 11.30 am
    - Modelling complex networks of over-densities and density voids.

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Presentation of numerical examples.

MODULE E: COSMOLOGICAL APPLICATIONS. (2 DAYS)

First Day – SaturdayFebruary 20, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - The null geodesic equation for LTB models. Computing observable quantities.

- Lecture 2: One hour - 10.30 am – 11.30 am
    - The spherical collapse model.

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Fitting observations: supenovae, CMB, BAO (review of literature)

Second Day - SundayFebruary 21, 2016

- Lecture 1: One hour
    - The null geodesic equation for Szekeres models. Computing observable quantities.
- Lecture 2: One hour - 10.30 am – 11.30 am
    - Inhomogeneous Hubble flow and peculiar velocities.

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Fitting observations: supenovae, CMB, BAO (review of literature)

MODULE F: OTHER EXACT SOLUTIONS. (2 DAYS)

First Day – MondayFebruary 22, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Non-tilted Bianchi models.

- Lecture 2: One hour - 10.30 am – 11.30 am
    - Tilted Bianchi models.

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Applications in Cosmology and other areas (review of literature)

Second Day - TuesdayFebruary 23, 2016

- Lecture 1: One hour - 9.15 am – 10.15 am
    - Perfect fluid shear-free solutions.
- Lecture 2: One hour - 10.30 am – 11.30 am
    - Shear-free solutions with anisotropic pressure and heat flux.

- Tutorial: Two hours - 11.30 am – 1.30 pm
    - Applications in Cosmology and other areas (review of literature)

The Faculty

P rofessor Roberto A Sussmanis a senior researcher in General Relativity and Theoretical Cosmology at the Institute of Nuclear Sciences of the National University of Mexico (ICN-UNAM (http://www.nucleares.unam.mx). He obtained his PhD in Queen Mary College, University of London, in 1987, under the supervision of **Professor Malcolm MacCallum**. His area of expertise is the study of inhomogeneous spacetimes through the properties of exact solutions of Einstein's equations, either in their application to Cosmology or in their usage to probe open theoretical problems in General Relativity. Professor Sussman has also worked in Aplications of Dynamical Systems in Cosmology, Relativistic Hydrodynamics and in Dark Matter models. He has publsihed more than 50 extensive articles in peer-reviewed journals (Classical and Quantum Gravity, Physical Revew D, Physics Letters B, General Relativity and Gravitation). He is a regular lecturer in Physics and Astronomy undergraduate and graduate courses and is also a frequent reviewer in peer-reviewed journals and in research proposals in Mexico, South Africa, Canada and Poland.

Professor Sushant G. Ghoshis aProfessor at Centre for Theoretical Physics, Jamia Millia Islamia, Delhi with over years of experience in Academic and Research. He is also serving as Joint Director (Research), Jamia MilliaIslamia, New Delhi.He was formerly in the Department of Mathematics at BITS Pilani Dubai, where he was also an In-Charge for grading activities. Before joining the BITS in 2003, Sushant served as Lecture and Reader in the Science College, Congress Nagar, Nagpur University, Nagpur since 1987. He is recipient of President of India's Best Research Award.

Sushant's teaching and research focuses on rotating black holes, general relativity, gravitational collapse and Astrophysics. Sushant is actively engaged in deriving exact solutions of black holes, and discuss different aspects of black holes in modified theories of gravity. He has published more than 60 research articles in peer-reviewed journals like Classical and Quantum Gravity, Physical Revew D, Physics Letters B, General Relativity and Gravitation, JHEP, EPJC etc..; He also reviewer for several top international Journals.

He is also an Honorary Professor at School of Mathematical Sciences in University of Kwazulu-Natal, South Africa since 2012, besides being an IUCAA, Pune associate since 1999.

Course Co-ordinator

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Course Registration Link:

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