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Title of the Thesis: A Study of Fixed Point Theorems in Hyperbolic Metric Spaces

Abstract

Due to its wide applications, fixed point theory is very fast growing research area of nonlinear analysis. The present thesis has been written for fixed point theory in hyperbolic metric spaces which are very useful, interesting, having convex structure (unlike Banach spaces or other linear spaces) and a geometrically rich nonlinear space. Efforts were made to contribute something to the subject which are well motivated. The thesis is divided into six chapters and each chapter is further divided into various sections, details of which are as follows: The thesis has been divided into six chapters. A brief summary of the research work carried out in this thesis is as:

Chapter 1 (Preliminaries): This chapter is introductory, here we describe basic definitions, formulae and results which are relevant to the subsequent chapters. Although most of these results are available in standard references on the subject, nevertheless we have collected them to make the thesis self-contained.

Chapter 2 (Few Iterations Involving Nonexpansive Mappings): This chapter is divided into three sections. In the first section, we study the convergence behaviour of Mann iteration in hyperbolic metric spaces for monotone nonexpansive mappings. In the second section, we study some strong and Δ-convergence for SP-iteration scheme in the hyperbolic metric spaces. In the final section, we extend semi-implicit midpoint rule for nonexpansive mappings in the setting of uniformly convex hyperbolic metric spaces and prove Δ-convergence and strong convergence.

Chapter 3 (On Monotone Nearly Asymptotically Nonexpansive Mappings): This chapter is also divided into three sections. In the first section, we prove existence theorems for fixed points of continuous monotone nearly asymptotically nonexpansive mappings in complete uniformly convex partially ordered hyperbolic metric spaces. In the second section, we study S-iteration

method to approximate the fixed points. In the third section, convergence results of Fibonacci Mann iteration scheme for monotone nearly asymptotically nonexpansive mappings in partially ordered hyperbolic metric spaces are presented. Also, establish the $w2-$ stability of Fibonacci-Mann iteration process.

Chapter 4 (Some Fixed Point Results Endowed with Graph): In this chapter, we study fixed point theorems endowed with graph. In the first section, we study some basic concepts of graph. In the second section, Browder and G¨ohde fixed point theorem for G-nearly asymptotically nonexpansive mappings is established. Some Δ-convergence and strong convergence results involing M-iteration for the same mapping also established. In the third section, we discuss common fixed point for semigroup of contraction mappings endowed with graph. Further, we give the existence of common fixed point for a semigroup of nearly asymptotically nonexpansive mappings in complete uniformly convex hyperbolic metric space endowed with graph.

Chapter 5 (Some Results on Multivalued Maps): In this chapter, we study fixed points for multivalued mappings. In the first section, we have studied endpoint which is smaller class than fixed points of multivalued mappings. We use multivalued version of M-iteration process to establish the convergence results of endpoints. The next section dealswith proximal point algorithm. In this second section, we present SP-iteration as PPA-SP-iteration for three multivalued nonexpansive mappings. We prove strong and weak convergence theorems for the proposed iteration process for minimizers of proper convex and lower semicontinuous functions and common fixed points of three multivalued nonexpansive mappings in Hilbert spaces.

Chapter 6 (A New Iteration Scheme Involving Nonexpansive Mappings): In the last chapter, we construct a new iteration process to approximate the fixed point for nonexpansive mappings. Also, we establish the stability of newly defined iteration process.