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Topic of Research : ELUCIDATION OF CERTAIN ASPECTS OF MULTIPLE HYPERGEOMETRIC FUNCTIONS

Finding

In the present thesis, several hypergeometric summation formulas are obtained and used to establish hypergeometric transformation and reduction formulas as well as their general double-series identities.

Chapter 1 focused on the brief history and progress of hypergeometric function.

In chapter 2, from sections 2.2 to 2.6, some interesting results are obtained for single and double hypergeometric functions, which may be potentially useful to non-specialist who are interested in Applied Mathematics or Mathematical Physics.

In chapter 3, we have proposed an alternative proof of Gauss's quadratic transformation by means of series rearrangement technique and suitable hypergeometric summation theorem for Clausen's series. Some special cases are also presented. Furthermore, we have generalized Gauss's quadratic transformation in terms of general double-series identity as well as a reduction formula for Srivastava-Daoust function.

In chapter 4, we have derived six general double-series identities. Moreover, by making use of these general double-series identities, we established six quadratic transformations for the product of exponential function and Goursat's function. We also established Kampé de Fériet's double hypergeometric functions.

In chapter 5, we have obtained four (presumably) new results for the terminating series 3F2[2]. Further, by the applications of these results, we established two general double-series identities and two quadratic transformations for Clausen's function. We also obtained two reduction formulas for Kampé de Fériet's double hypergeometric function. Some known and new results are also obtained as special cases of our findings.

In chapter 6, we have derived certain generalizations of Kümmer's second summation theorem (1.7.3) for Clausen's hypergeometric series 3F2[1/2] in section 6.2 when one numerator and one denominator parameter are differing by a positive integer. Some known and new results are also presented as special cases of our main findings.

In chapter 7, we have derived a summation formula for the terminating Clausen's series 3F2[1]. As the applications of our new summation formula we established two linear transformations for 4F3[z] and 3F3[z], and a reduction formula for Kampé de Fériet's double hypergeometric function with arguments z and -z. We also constructed a general double-series identity.

In chapter 8, We have obtained a summation formula for the terminating Clausen's series 3F2[1]. Furthermore, by using our new summation formula we established a reduction formula for Srivastava-Daoust function. We also derived a general double-series identity involving a bounded sequence of complex numbers.

In chapter 9, we have derived a summation formula for the series 4F3[1] in which two numerator and two denominator parameters are differing by unity. By means of our summation formula we established a general double-series identity, two linear transformations for 4F3[z] and 3F3[z]; and a reduction formula for Kampé de Fériet's double hypergeometric function with arguments z and -z.

Summary of Abstract

The primary objective of this work is to investigate the hypergeometric functions 2F1, 2F2, 3F2 and 4F3 with the suitable arguments and develop new hypergeometric summation, transformation, and reduction formulas. We also

generalize these hypergeometric formulas in terms of general double-series identities. In particular we give an another proof of Gauss's quadratic transformation with it's generalizations.

The Special functions appear as solutions of different classes of mathematically and physically relevant functional equations. The solution of these equations were found to be some new functions which possessed interesting properties like orthogonality, integral transformation, differentiation, convergence etc. These functions were named as special functions. The most important classes of special functions are gamma function, beta function, hypergeometric functions, confluent hypergeometric functions, Bessel functions, Legendre functions, parabolic cylinder functions, integral sine and integral cosine functions, incomplete gamma functions and incomplete beta functions, probability integrals; various classes of orthogonal polynomials in one or several variables; elliptic functions, elliptic integrals, Lamé functions, Mathieu functions, the Riemann zeta function, automorphic functions and some special functions of a discrete argument.

In the present work, an attempt has been made to study the "ELUCIDATION OF CERTAIN ASPECTS OF MULTIPLE HYPERGEOMETRIC FUNCTIONS".
This thesis is spread over in ten chapters.

At the end of the thesis, a bibliography is arranged, which by no means is exhaustive on the subject. In fact, only those works have been listed which have been referred in the body of the thesis.