Regular Black Holes in Lovelock Gravity and their Thermodynamic Properties

ABSTRACT

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Abstract

Einstein's theory of general relativity has been the most successful theory of gravity that explains the gravitational dynamics through the geometry of spacetime. One of the most celeberated theoretical prediction of general relativity is the existence of the ultimate sponges of nature, the black holes. The black holes in GR suffers from singularity problem, where the theory looses its predictive power. One of the solution to the singularity problem is the introduction of regular black hole models, which generates the outward radial pressure at center ($r \to 0$) and prevents the formation of singularity. Hence, in recent times the regular black holes have attracted a lot of attention. Initially, the black holes solutions were discussed only in $4D$ spacetime, but in last few decades gravity witnessed a considerable activity in higher dimesnions motivated by the superstring and field theories. In addition to higher-curvature corrections to Einstein theory, string theory makes several predictions about nature, the most important ones are the existence of extra dimensions. The Einstein-Gauss-Bonnet gravity is a natural and most effective generalization of Einstein's general relativity, to higher dimensions, motivated by the heterotic string theory. The Einstein-Gauss-Bonnet (EGB) theory allows us to explore several conceptual issues of gravity in a broader setup and the theory is known to be free of ghosts while expanding about the flat space. The effective field equations, in the EGB theory, are of second-order like in gen-

eral relativity, but admit new black hole solutions in $D \geq 5$ spacetime that are unavailable to the Einstein theory. Motivated by this fact, we work on black hole solutions in EGB gravity and their thermodynamic properties.

In this doctoral thesis, we discuss some black holes in Anti de Sitter (AdS) spacetime in the scope of GR and EGB gravity and also dicuss their thermodynamical behaviour through varios thermodynamic quantities like, Hawking temperature, entropy, specific heat and free energy. The thermodynamics of black holes in AdS spacetime is extremely important due to the AdS/CFT correspondence. While studying the black hole thermodynamics in AdS spacetime, the negative cosmological constant Λ is considered as the positive pressure P and its conjugate as the thermodynamical volume V of the black hole. This consideration leads to the modification of first law of black hole thermodynamics through the addition of "PdV" term.

Chapter 1 is introductory in nature, that begins with a brief introduction to Einstein's general relativity and black hole solutions in the scope of GR. Then, we discuss Reissner-Nordström black holes and their thermodynamics in AdS spacetime. Next, we talk about the importance of regular black holes and introduce the Bardeen and Hayward black holes in brief. Then, we discuss the motivations for modifications in GR and also give a brief introduction of EGB gravity. We also discuss the the Boulware-Deser black with their thermodynamics. Finally, we present the future scope of our research work.

In Chapter 2, we obtained an exact black hole in EGB gravity for a static and spherically symmetric D-dimensional AdS spacetime with energy-momentum given by nonlinear electrodynamics. The solution exhibits horizons which could be at the most two. Later, we compute exact expressions for Hawking temperature, entropy, heat capacity, and free energy associated with the black holes, also demonstrate that Hawking-Page transition is achievable. We perform a detailed analysis of the thermodynamical specific heat with a focus on local and global stability. It turns out that heat capacity can be negative or positive depending on the choice of parameters e and α, which further, respectively, tells us that the black hole is unstable or stable.

Indeed, the phase transition of a black hole is characterized by the divergence of its specific heat at a critical horizon radius r_+^C which is varying with the spacetime dimension D and parameter α. The black holes are thermodynamically stable with a positive heat capacity for the range $r_1 < r_+ < r_+^C$ and unstable for $r_1 > r_+ > r_+^C$. We discussed the black hole remnant and tabulated the numerical values of black hole remnant size and mass. The results presented here are the generalization of the previous discussions and in the appropriate limits, go over to AdS-EGB black holes and EGB black holes. The possibility of a further generalization of these results to Lovelock gravity is an interesting problem for future research.

Chapter 3 is devoted to obtain exact Hayward-like regular black holes in EGB coupled to nonlinear electrodynamics. The solution has an additional parameter charge e due to nonlinear electrodynamics, apart from the black hole mass (M). The previously known case like the famous Boulware-Desser black holes of EGB theory is encompassed as a special case. In turn, we characterized the solution by analyzing horizons which are a maximum of two, viz. Cauchy, and Event horizons. The regularity of the spacetime is confirmed by calculating various curvature invariants and shown to be well behaved everywhere including at origin. We have also computed thermodynamical quantities like Hawking temperature, entropy, specific heat, and free energy associated with $5D$ EGB-Hayward black holes with a focus on the stability of the system. It is demonstrated that specific heat diverges at horizon radius r_C which incidentally corresponds to the local maximum of the Hawking temperature at r_C. It is shown that the specific heat is positive in the region $r_1 < r < r_C$, which signifies that the small black holes are thermodynamically stable against perturbations in the region, and the phase transition exists at r_C. The black hole is unstable for $r_1 > r > r_C$. The global stability analysis of black holes is also done by calculating free energy. Besides, we have also shown that after black hole evaporation there will be a stable remnant with zero temperature and positive specific heat.

In Chapter 4, we have obtained exact nonsingular black holes in AdS spacetime which encompasses the Schwarzschild-AdS as a special case in the absence of nonlinear electrodynamics ($k = 0$) and asymptotically ($x \gg \tilde{k}$) mimics the well known Reissner-Nordstrom-AdS black hole. Thus, we have constructed exact spherically symmetric, static, regular black hole metrics in the context of general relativity minimally coupled to nonlinear electrodynamics theory. The new nonsingular-AdS black hole metric is characterized by horizons which could be at least two describing a variety of self-gravitating objects, including an extremal black hole with degenerate horizons and non-extremal black holes with two distinct horizons. We have shown that the exponential correction term arising due to nonlinear electrodynamics can cure the singularity problem and analyze in the detailed thermodynamical properties of the obtained nonsingular-AdS black hole metric were investigated. We considered that the cosmological constant plays the role of the positive pressure of the system. After having calculated the desired thermodynamic variables, we discussed the thermodynamical quantities like the Hawking temperature, entropy, specific heat at constant pressure, and Gibb's free energy in extended phase space. The resultant forms of the specific heat and the Gibbs free energy indicate a first-order phase transition between small/large black holes. The P vs r_+ diagram shows that below the critical pressure P_c, the nonsingular-AdS black hole undergoes phase transition between small/large black holes reminiscent of the liquid/gas coexistence. Finally, we discussed the $P - V$ criticality for our nonsingular-AdS black hole, and by analyzing the behaviour of various thermodynamical quantities near the critical point, we found the values of the critical exponents and showed that these values are the same as those for the Van der Waals gas.

In Chapter 5, we have presented exact D-dimensional Hayward-like black holes in Einstein-Gauss-Bonnet gravity with a negative cosmological constant, thereby generalizing Boulware-Deser black holes which are included as a special case ($e = 0$, $l^2 \to \infty$). The Hayward-EGB-AdS black holes are characterized by analyzing horizons, which at most could be two, viz. inner Cauchy and outer event horizons. In turn, we have analyzed the horizon thermodynamical properties and phase structure of these AdS black holes. Despite complicated solutions, we obtain the exact expression for the thermodynamical quantities like the black hole mass, Hawking temperature, entropy, and free energy at event horizon r_+ and in turn, we also analyze the thermodynamical stability of the black holes by studying the specific heat. The entropy of the black holes is modified due to the magnetic charge e and the GB parameter α, and area law $S = A/4$ is no longer valid. The phase transition is detectable by the divergence of the heat capacity (C_+) at a critical radius r_c (changes with e, α, l^2 and dimensions D), such that the black hole is stable in the region viz: $r_{t1} < r_+ < r_C$ and $r_+ > r_{t2}$ with positive heat capacity ($C_+ > 0$), on the other hand the heat capacity is negative ($C_+ < 0$), when $r_+ < r_{t1}$ and $r_c < r_+ < r_{t2}$, indicating the instability of black holes. Interestingly, the smaller and larger Hayward EGB-AdS black holes are stable with ($F_+ < 0$). In contrast only smaller Hayward and only larger EGB-AdS black holes ($r_+ \gtrsim 5.6$ in $5D$) are globally stable with negative free energy. Finally, Hayward EGB-AdS black holes do not completely evaporate, but halts into a stable remnant with vanishing temperature, degenerate horizons and positive heat capacity $C_+ > 0$. The black hole remnant size and mass for different values of the black hole parameters are also given. Finally, we recovered all results of EGB-AdS/EGB black holes in the limits $e = 0/e = \Lambda = 0$ and that of the general relativity when $\alpha \rightarrow 0$.

In Chapter 6, we obtained static spherically symmetric Bardeen black hole solution in the EGB gravity theory in 4-dimensional spacetime by re-scaling the GB coupling constant $\alpha \rightarrow \alpha/(D-4)$ at the level of the equation of motion. The obtained solution is sourced by the NED field and belongs to the class of three-parameter family of static and spherically symmetric black holes. We analyzed the black hole horizon structure and find that for a fixed value of GB coupling constant α and magnetic monopole charge g, there exists a minimal value of mass, M_0, below which no black hole solution exists. Whereas, for $M = M_0$ black hole possess degenerate horizons, and for $M > M_0$ two distinct horizons. Similarly, we find the extremal values of $g = g_0$ and $\alpha = \alpha_0$ for which black hole admits degenerate horizons $r_- = r_+ = r_E$.

We made a detailed analysis of Helmholtz free energy F_+ and heat capacity C_+ to check the thermodynamical stability of the black holes. We found out that Bardeen black holes in 4D EGB gravity are globally thermodynamical unstable as the Helmholtz's free energy is always positive. While analyzing the local thermodynamical stability through specific heat C_+, we found that there exists a critical value of horizon radius r_+^c at which black holes show second-order phase transition with diverging specific heat. The black holes with $r_+ < r_+^c$ are locally stable with $C_+ > 0$, whereas the black holes with $r_+ > r_+^c$ having negative specific heat are locally unstable.

Chapter 7 includes an exact 4D nonsingular EGB black hole metric, characterized by three parameters, mass (M), the Gauss-Bonnet coupling constant (α) and magnetic monopole charge (g), and it regains the 4D EGB metric as a particular case in the absence of magnetic charge ($g = 0$).

The analysis of horizon structure leads us to get a maximum allowed value of Gauss-Bonnet coupling constant, α_0, for fixed values of M and g such that for $\alpha > \alpha_0$ no black hole solution. Whereas, $\alpha < \alpha_0$ and $\alpha = \alpha_0$, respectively, corresponded black hole with double and single degenerate horizon radius. We computed the Hawking temperature (T_+), entropy (S_+), heat capacity (C_+) and Gibb's free energy (F_+) associated with the horizon radius of the black hole. In turns, we in details analyzed the specific heat and found that there existed second-order phase transition with diverging C_+ at critical radius, r_+^c. The black holes with $r_+ < r_+^c$ found to be locally thermodynamical stable and on the other hand black holes with $r_+ < r_*$ and $r_+ > r_+^c$ found to be locally unstable. While the analysis of Gibb's free energy F_+ leads us to find that $4D$ nonsingular EGB black holes with smaller horizon radius are globally stable with $F_+ < 0$, whereas those with the more considerable value of horizon radius having $F_+ > 0$ are globally unstable. Finally, we have also shown that the $4D$ nonsingular EGB black holes evaporation results in a stable black hole remnant with zero temperature $T_+ = 0$ and positive specific heat $C_+ > 0$.

Finally, we conclude the thesis by summarizing the outcomes of our work in Chapter 8.