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ABSTRACT

Bose-Einstein condensate (BEC) of alkali atomic vapours in a harmonic trap is an important subject in many-body quantum systems with interparticle interaction. The decisive experimental control and precision offered by atomic physics, makes these ultra-cold systems an outstanding one to study the novel quantum mechanical phases. Towards this goal, in this thesis, we present the exact diagonalziation study of interacting BEC with finite-range Gaussian repulsion (instead of widely used δ-function potential) in quasi-two-dimensional harmonic trap subject to an external rotation. We investigate the may-body ground state as well as low-lying collective excited states to examine the quantum mechanical novel phases in Bose-condensed gas. We mainly focused on the finite model systems in which one deals with a few number of atoms, so that the exact diagonalization of the Hamiltonian matrix can be carried out in subspaces of quantized total angular momentum L_z using Davidson iterative algorithm. Our approach involves the inclusion of lowest as well as higher Landau levels with single-particle angular momentum of either sign for the construction of many-body wavefunction. In order to analyze the many-body quantum state properties of the rotating condensate, we calculate the first order correlation as embodied in single-particle reduced density matrix. The degree of condensation and von Neumann entanglement entropy is calculated as a measure of many-body quantum correlation, for quantum mechanically stable as well as unstable states in the co-rotating frame. To gain an insight into the many-body states, the conditional probability distribution is analyzed for the internal structure of the eigenstates.

In response to externally impressed rotation, the patterns of singly quantized vortices are formed, shaping into polygonal vortex patterns with and without a central vortex at the trap center. The stable polygonal vortex patterns having discrete p-fold rotational symmetry with $p = 2, 3, 4, 5, 6$, are observed. The hexagonal vortex pattern with $p = 6$ symmetry, is a precursor to the triangular vortex lattice of singly quantized vortices in the thermodynamic limit. Our exact diagonalization result on a finite system, bears the signature of the thermodynamically stable triangular vortex lattice composed of singly quantized vortices. The internal structure of unstable states reveals the mechanism of entry, nucleation and pattern formation of vortices with structural phase transition, as the condensate goes from one stable vortical state to the other. For unstable states, quantum melting of vortex patterns due to uncertainty in positions of individual vortices, is also briefly discussed.

Our numerical results show that breathing modes with N-body eigenenergy spacing of $2\hbar\omega_\perp$, known to exist in strictly 2D system with zero-range (δ-function) interaction potential, may as well exist in quasi-2D system with finite-range Gaussian interaction potential. In the rapidly rotating regime for three boson system, the ground state in angular momentum subspaces $L_z = \frac{q}{2}N\left(N-1\right)$ with $q = 2, 4$ is found to exhibit the anticorrelation structure, suggesting that it may variationally be described by a Bose-Laughlin like state. We further observe that the first breathing mode exhibits features similar to the Bose-Laughlin state in having eigenenergy, von Neumann entanglement entropy and internal structure independent of interaction for the three-boson system considered here. On the contrary, for eigenstates lying between the Bose-Laughlin like ground state and the first breathing mode, the values of eigenenergy, von Neumann entropy and internal structure are found to vary with interaction. Having analyzed the few body case, we have also studied the breathing mode collective excitations for a rotating BEC of of many N-boson system (in the vicinity of first central vortex state with $L_z = N$). For a given L_z, the low-energy eigenspectra (bands) are obtained in weakly to moderately interacting regime. Further, for a given interaction, the split in low-lying eigenenergies with increasing L_z, is the precursor to spontaneous symmetry breaking of the axisymmetry associated with the entry of the first vortex. With increase in repulsive interaction, the value of the first breathing mode increases for stable total angular momentum states $L_z = 0$ and N, but decreases for intermediate $0 < L_z < N$ metastable states. The ordinal position of the observed first breathing modes in the eigenspectrum remains unchanged as the interaction is varied over several orders of magnitude.

We have also studied the finite-range effect of the repulsive Gaussian interaction (with large s-wave scattering length), on the many-body ground state and low-lying excited states of non-rotating as well as rotating system of spinless bosons. The study ranges from the few-body ($N = 2$) system to the many-body ($N = 16$), system where quantum (Bose) statistics becomes perceptible. In particular, we analyze the effect of interaction range of the Gaussian potential on the ground state energy of different L_z states as well on the critical angular velocity $\Omega_{\mathbf{c1}}$ of first vortex ($L_z = N$) state. For an optimal value of the range of the Gaussian interaction potential, the nucleation of the first centered vortex may begin at a lower value of the rotational angular velocity as compared to the zero-range interaction potential. Moreover, we explore the role of interaction potential range on the quantum correlation (measured in terms of the degree of condensation and the von Neumann entropy), for small systems ($N = 2$ to 8) where an understanding of few-body effects provide valuable insight into large systems ($N = 8$ to 16) which admit the possibility of many-body correlation. The results obtained indicate that the finite-range Gaussian interaction potential enhances the degree of condensation compared to the zero-range interaction potential. Our theoretical results may be relevant for experiments currently conducted on quasi-two-dimensional Bose gas with more realistic interaction potential.