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Topic: Shortest Path Algorithms in Multigraphs & RT-Multigraphs

ABSTRACT

It is observed by us that there are many real life cases especially in Network Systems where the classical notion of Graph Theory is not well applicable. It is because of the fact that the network under consideration cannot be modeled into graphs but can be well modeled into multigraphs. Those situations cannot be dealt with the theory of graphs, but by the theory of multigraphs only. We observed that the notion of Multigraphs can be the most useful mathematical tool in such Networks, where the classical theory of graphs fails. Therefore there is a very genuine need to cater solutions to the real issues of communication or transportation problems on these type of networks by the 'Theory of Multigraphs' or by a new and advanced type of graphs of higher potential, instead of the classical traditional graphs of existing literature. This is the main philosophy which knocked our mind to initiate a rigorous research work for the Ph.D. thesis in this thrust area.

There is not much work done or reported in the existing literature so far by the world scientists on the 'Theory' of Multigraphs. Consequently, our very initial contribution in this thesis (one chapter is devoted) is made by doing a rigorous work on the 'Theory of Multigraphs', especially on unearthing their important properties and characterizations, on various useful operations on them, on algebraic development of the theory of multigraphs, on various interesting propositions on the multigraphs with several real life examples, etc. Multigraph (Graph) is also an important non-linear data structure in Computer Science. A multigraph (graph) can have infinite number of vertices and edges. However, in this thesis work it is assumed that the number of vertices and edges are both finite.

Many of the problems on cost minimization in communication network are being solved using the famous Dijkstra's algorithm of graph theory, not on the theory of multigraphs. Since graphs are the very special cases of multigraphs, the theories, results, algorithms, etc. of the theory of graphs cannot be automatically accepted to be valid in the theory of multigraphs. Consequently, the next important problem unearthed before us is "to design an algorithm to solve SPP in multigraphs with the philosophy of the scientist Edsger Dijkstra used by him in his path breaking algorithm". This is done in this thesis work.

In real world, the cost of a path in a graph is not always crisp because of its imprecise nature, and consequently for such cases it can be considered to be an intuitionistic fuzzy cost (or fuzzy cost). Consequently, the famous classical SPP problem in a generalized way has been studied by many authors as "Fuzzy Shortest Path Problem"(FSPP) or in a more generalized way as "Intuitionistic Fuzzy Shortest Path Problem" (IFSPP). But the algorithms for FSPP or ISPP of graphs cannot be applicable to multigraphs. Consequently, the next important problem unearthed before us is "to design an algorithm to solve IFSPP or FSPP in multigraphs". Our basic objective is to provide solution to the SPP of networks which cannot be modeled into graphs but multigraphs only, even if the costs are vague parameters. This is done in this thesis work.

In many real situation, few or many or all of the existing links/arcs of a given network (graph or multigraphs) are not always in their capable condition for communication at a real instant of time, because of the reason that these links may be sometimes disabled or blocked temporarily and waiting for maintenance/repair; Consequently, the mathematically calculated shortest path extracted by using classical Dijkstra's algorithm may become failure in practical. Consequently, there is a need to develop an alternative way so that communication remains uninterrupted and also 'real time optimal' at that real instant of time, although not theoretically the absolutely optimal. This is done in this thesis work.

We then consider a further realistic situation with more generalized thoughts. In the previous problem we considered the real time situation that one link is either capable or non-capable of communication at some real instant of time, i.e. in the two-valued logical status. But after that a sudden knock of Prof. Zadeh's philosophy generated a new interesting problem to us. Although the cost of each link is fixed in a given graph (multigraph), but condition of communication capability via a link may be viewed in the range of some index in the closed unit interval [0,1]. The present day networks of communication systems, be it a giant or not, contain a lot of uncertainties, in particular regarding attack or damage from internal or external sides. Thus, in many real situation, few or all of the existing links/arcs of a given network or multigraph/graph are not always in their original condition (for communication) physically or logically, rather in a weaker condition at a real instant of time, or even sometimes partially or fully disabled/blocked temporarily and waiting for maintenance/repair; finally causing delay in communication or transportation. Although a multigraph/graph has a constant topology with pre-defined weights/costs of its links, but with real time situation the link status of each link may be changing from their original values. Consequently, the mathematically calculated shortest path extracted by using classical Dijkstra's algorithm or our developed generalized Dijkstra's algorithm may become costlier in terms of time and/or in terms of other overhead costs; whereas some other path may be the most efficient or most optimal at the real time under consideration. This leads us to think of a very useful problem in communication network on how to view the multigraphs (or graphs) in a better and appropriate way compared to the classical pattern and then how to develop new algorithms, appropriate results considering the real time situation, not considering the original theoretical situation. This is done in this thesis work and we claim that this work could lead to useful solutions to real time problems of present and future communication network models.

The Thesis is organized into eight chapters in total. Details are given below:

Chapter 1 is on an overall Introduction of the complete work reported in this Thesis.

Chapter 2 very briefly describes the preliminaries from the existing literatures about Graphs & Multigraphs, fuzzy theory and intuitionistic fuzzy theory for soft computing, about the classical SPP (Shortest Path Problem), FSPP (Fuzzy Shortest Path Problem), IFSPPP (Intuitionistic Fuzzy Shortest Path Problem).

Chapter 3 contains a good amount of theoretical characterization of the data structure "Multigraph" (being a generalized model of graph) for modeling communication network.

Chapter 4 contains a modified algorithm as the generalization of the classical Dijkstra's Algorithm for extraction of shortest path in a crisp multigraph, with applications.

Chapter 5 describes methods to find out intuitionistic fuzzy shortest path in a Multigraph & Graph, developing corresponding algorithms and results.

Chapter 6 introduces new models called by RT-Multigraphs (Real Time Multigraphs) & RT-Graphs (Real Time Graphs) for Communication Networks, and describes solution of SPP in real time (RT) environment.

Chapter 7 introduces new models called by GRT-Multigraphs (Generalized Real Time Multigraphs) & GRT Graphs (Generalized Real Time Graphs) for Communication Networks, and also about the development of algorithms to solve SPP in GRT environment.

Chapter 8 contains an overall 'Conclusion' of the complete work; and Future Scope of work in our research direction with respect to the newly introduced models: RT-multigraphs and GRT-multigraphs.