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TITLE OF THE THESIS: **A Study of Wavelet Packets and Non-Uniform Wavelet Packets**

### ABSTRACT

**Keywords:** *wavelet, wavelet packet, multiresolution analysis, Hardy Littlewood maximal function, Hardy Space, dimension function, Radial decreasing  $L^1$ -functions.*

The main aim of this thesis is to extend some well known results of wavelets to wavelet packets and to non-uniform wavelet packets and to study the properties of the wavelet packets and non-uniform wavelet packets.

In the present thesis, we have carried out a systematic study of wavelet packets and non-uniform wavelet packets. The thesis as such has been classified into four chapters.

In Chapter-1, we have studied application of dimension functions of wavelet packets. By using above characterization we have proved that there is no orthonormal wavelet packet for  $\mathcal{H}^2(\mathbb{R})$  satisfying the regularity condition.

Recently, Behera in [8] gave the construction of nonuniform wavelet packets associated with nonuniform multiresolution analysis. Ahmad et al. [6] have studied convolution bounds and convergence of wavelet packet series.

In Chapter-2, we have studied the convolution bounds of nonuniform wavelet packets. We also studied that the convolution bounds of kernel  $Q_j^n(x, y)$  defined as  $Q_j^n(x, y) = \sum_{\lambda \in \Lambda} \omega_{j,n,\lambda}(x) \overline{\omega_{j,n,\lambda}(y)}$  where  $\omega_{j,n,\lambda}(x)$  is a nonuniform wavelet packet.

The problem of convergence of the wavelet series has been studied by Meyer [36], Walter [44, 45], and Kelly et al. [31, 32]. Recently, Ahmad and Kumar [3] have extended the results of Kelly *et al.* [31, 32] to the stationary wavelet packets and have shown that wavelet packets expansion of any  $L^p$ -function ( $1 \leq p \leq \infty$ ) converges pointwise almost everywhere under certain conditions. On the otherhand, Nielson [37, 38] introduced the non-stationary wavelet packets generated by the Haar filters which he called them *Walsh-type wavelet packets* for which the same type of  $L^p$ -convergence results hold. Motivated and inspired by the importance of nonuniform wavelet packets,

In Chapter-3, we have studied the pointwise convergence of nonuniform wavelet packet series by assuming that the nonuniform wavelet packets being used be bounded by radial decreasing  $L^1$ - functions.

In Chapter-4, we have proved the results on the existence of unconditional nonuniform wavelet packet bases for spaces  $\mathcal{H}^1(\mathbb{R})$  and  $L^p(\mathbb{R})$ ,  $1 < p < \infty$  based on the approach similar to that of Meyer [36] and Coifman [17].