

Name of Scholar: Moin Akhtar Ansari
Name of Supervisor: Prof. Mohd. Rais Khan
Name of Co- Supervisor: Prof. V. N. Dixit
Department: Mathematics
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ABSTRACT

In the present thesis entitled "Quasi-ideals in Algebraic System", we have studied quasi-ideals, (m, n) quasi-ideals, bi-ideals and (m, n) bi-ideals in different algebraic structures viz. semigroups, quotient semigroups, involution semigroups, regular semigroups, Γ -semigroups, po-semigroups, po- Γ -semigroups, regular po- Γ -semigroups and involution rings etc.

Chapter 1 consists of the basic definitions and essential results which are used throughout the thesis. Quasi-ideals for semigroups and rings have been initiated and developed by Steinfeld ([70], [71], [72] and [73]). Further it has been investigated by many authors e.g. Stewart [74], Szasz [75], Weinert [77]. We have used these definitions and results in the subsequent chapters to serve the purpose to acquaint the readers with the terminology and basic facts.

Chapter 2 deals with (m, n) quasi-ideals in semigroups and regular semigroups. Chinram [17] has studied (m, n) quasi-ideals in rings. Further Chelvam [13] has given the concepts of generalized (m, n) bi-ideals of a near ring. Tilidetzke [76] has shown a characterization of a $\mathbf{0}$ -minimal (m, n) ideals in semigroups. Krgovic ([42] and [43]) has proved results based on bi-ideals and $(0, 2)$ bi-ideals in semigroups. All these led us to give the concept of (m, n) quasi-ideals in semigroups and regular semigroups etc. In the last section of this chapter we have included the concept of ordered (m, n) quasi-ideals and ordered (m, n) bi-ideals in po-semigroups.

Chapter 3 gives the concept of (m, n) bi- Γ -ideals in Γ -semigroups. We have observed that most of the properties of bi- Γ -ideals in Γ -semigroups are analogous to (m, n) bi- Γ -ideals in Γ -semigroups.

Chapter 4 relates the roughness of (m, n) quasi-ideal in semigroups and their extension in Γ -semigroups. Also we study μ -lower and μ -upper rough (m, n) quasi-ideals in semigroups, quotient semigroups, Γ -semigroups and quotient Γ -semigroups.

Chapter 5 elaborates the generalized (m, n) bi-ideals in semigroups with involution. The purpose of this chapter is to give some properties on generalized (m, n) bi-ideals in semigroups and semigroups with involution. We have also taken semiprime, prime and strongly prime generalized (m, n) bi-ideals in semigroups under consideration and observed that semiprime, prime and strongly prime generalized (m, n) bi-ideals in semigroups are related to each other in certain order.

Chapter 6 states the condition under which a maximal quasi-ideal in a ring will become $*$ -maximal quasi-ideal in rings with involution. We have derived some results which show the relation between quasi-ideals in ring to that of the $*$ -quasi-ideals in $*$ -rings and likewise bi-ideals in rings to that of $*$ -bi-ideals in $*$ -rings.