ABSTRACT OF THE THESIS ON

THE STUDY OF CERTAIN MULTIVARIABLE GAUSSIAN HYPERGEOMETRIC SERIES

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Multiple hypergeometric functions constitute a natural generalization of the Gauss hypergeometric functions of one variable. Since the introduction of double hypergeometric function by Appell and triple hypergeometric functions by Lauricella, numerous papers by many workers have been published and the theory has been considerably extended. An extensive study has been made in Europe, America and India for these multiple functions, which produced explosion of knowledge of the subject.

Chapter 1 contains the definitions, notations of special functions and multiple hypergeometric functions with their convergence conditions. This chapter is intended to provide an introduction to variety of hypergeometric functions along with other special functions used in the thesis.

In the literature of special functions, the summation theorems related with single Gaussian hypergeometric functions play an important role in the study of transformation and reduction formulae of multiple Gaussian hypergeometric functions.

The aim of the present chapter is to introduce several classes of special functions, which occur rather more frequently in the study of summations, transformations and generating functions needed for the presentation for subsequent chapters. Section 1.1, gives the equivalent definitions of Gamma functions, Digamma function, Polygamma function, Pochhammer symbol and an important lemma (1.1.23). in section 1.2, the definition of Gaussian hypergeometric function ${}_2F_1$ and their generalization with their convergence conditions, Generalized ordinary bilateral hypergeometric series and truncated Gaussian hypergeometric series have been discussed. Sec 1.3 describes the definition of Wright generalized hypergeometric function and their convergence conditions. A brief account of hypergeometric functions of two and several variables are presented in sections 1.4 to 1.10. Section 1.11, presents series manipulations, based on certain interchanges of the order of summations of a double (or multiple) series. In sections 1.12 and 1.13, transformations and summations theorems for hypergeometric series, definition of Catalan's constant, Riemann's zeta function and polylogrithms, are given. The definition of classical hypergeometric polynomial is given in section 1.14. in section 1.15, some theorems and integrals of Ramanujan are discussed.

The main objective of **Chapter 2,** is to obtain some new hypergeometric summation theorems associated with ${}_2F_1\,(\pm1)$, ${}_3F_2\,(\pm1)$, ${}_4F_3\,(\pm1)$, ${}_5F_4\,(\pm1)$, ${}_6F_5\,(\pm1)$ for specific values of numerator and denominator parameters, which are not available in the literature on special functions, by means of Ramanujan series, Berndt series and Sanjana series so far.

Section 2.2 is associated with known hypergeometric summation theorems obtained from certain Ramanujan series. Section 2.6, 2.7 and 2.8 discuss new hypergeometric

summation theorems obtained from more Ramanujan series, Sanjana series and Bruce Berndt series, which are given in the sections 2.3, 2.4 and 2.5 respectively. In section 2.9, derivations of new hypergeometric summation theorems, are given with the help of the lemma (1.1.23) and series iteration techniques.

Chapter 3 deals with hypergeometric forms of different elliptic type integrals. It has been studied because of its importance and application in certain problems involving computations of the radiation field off axis from a uniform circular disc radiating according to an arbitrary distribution law.

In this chapter, we have established Gaussian hypergeometric forms of complete elliptic integral of third kind **II**(*k*, *a*), Andrews integral, Erdélyi's complete elliptic integrals B(k), C(k) and Wright's hypergeometric forms ${}_2\Psi_1$ of some new elliptic type integrals in the forms of

$$\int_0^{\pi/2} \frac{\sin^M \theta \, d\theta}{(a^2 + b^2 + c^2 - 2ab \sin^N \theta)^g} \quad \text{and} \quad \int_0^{\pi/2} \frac{\sin^M \theta \, d\theta}{(a^2 + b^2 + c^2 - 2ab \cos^N \theta)^g}$$

The main aim of the **Chapter 4** is to obtain some new as well as known generating functions for even and odd hypergeometric polynomials of Madhekar – Thakare, Konhauser, Jacobi Laguerre. This chapter is in continuation with earlier works given in the literature. The proof of generating relations for the polynomials of Madhekar and Thakare has been given here.

In **Chapter 5**, the correct forms of Slater's Mellin-Barnes type contour integrals of the products of Bessel's functions, Kummer's confluent hypergeometric functions and Gauss's ordinary hypergeometric functions, using Whipple's theorem have been obtained and presented.

The **Chapter 6** is devoted to the investigation of a general theorem (6.2.2) and additional relations (6.3.1), (6.3.2), (6.3.3) and (6.3.4), using series rearrangement techniques. Its special cases yield various generating relations for Dattoli, Torre, Lorenzutta, Qureshi, Yasmeen, Pathan, Panja-Basu, and Rainville which are given in section 6.4.some generating relations for Dattoli type polynomials are obtained, which are closely related to generalize Hermite polynomials. Some new and known generating functions follow as special cases of our main results. Further, some generating relations associated with Srivastave-Daoust multiple hyper geometric functions have been derived.

In **Chapter 7**, some interesting truncated Gaussian hypergeometric summation theorems (7.2.1) to (7.2.10) have been obtained using Euler's identity (7.1.2) and de-Moivre's theorem associated with the compact forms of the factors (7.1.3) to (7.1.10) whose arguments, numerator and denominator parameters are real complex numbers. A known problem (7.3.5) of S. Ramanujan, solved by K. R. Rama Aiyar and K. Appukuttan Erady is also obtained as a special case of our findings. The proof of the following lemma (which has an application in section 7.3) is presented in this chapter.

In **Chapter 8**, some hypergeometric reduction formulae for ${}_3F_4$, ${}_4F_5$, ${}_5F_6$, and ${}_6F_7$ (whose numerator and denominator parameters are real complex numbers and argument is $x^2/4$), have been obtained. By means of mathematical induction, finite difference calculus and series iteration technique, the alternative proofs of useful theorems (8.3.1), (8.3.3), Montmort theorem (8.3.5) and S. Narayana Aiyar theorem (8.3.6) have been established and given in section 8.3. In section 8.4, by means of de-Moivre's theorem, Cardan's method, Ferrari's method and Descarte's method, some typical linear factors of polynomials of degree three, four, five and six, are given. In section 8.5, Ordinary forward differences of different order for some polynomial functions are given, which are very useful in present study. Derivations of main reduction formulae (8.2.1) to (8.2.12) are discussed in section 8.6.

Finally study concluded with the interaction and connection between the hypergeometric functions of different types and nature; contemporarily demonstrating that multiple hypergeometric series possesses a special character and particular applications.