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Topic of Research: "Dynamical Analysis and Synchronization on Nonlinear Chaotic Systems"

Findings

Chaotic phenomenon in nonlinear dynamical system is characterized by complexity, irregular behavior, unpredictability, and highly sensitivity to initial conditions. The primary objective of the thesis is to conduct a thorough dynamical analysis and explore the synchronization of chaotic systems, demonstrating how two or more chaotic systems can align their behaviors through effective coupling or external control under appropriate conditions. The study examines the properties such as phase-space trajectories, Poincaré maps, bifurcations, Lyapunov exponents, and competitive modes. It also delves into the profound insights through Hamilton energy theory and boundedness concepts. This not only deepens the understanding of chaotic systems but also facilitates the development of methods for controlling, forecasting, and harnessing chaos across diverse scientific and engineering disciplines.

Chapter 1 provides an overview of chaos theory, along with a discussion of chaotic properties of the systems in different environments and circumstances. It also covers various techniques related to the dynamical analysis of chaotic systems. Moreover, fundamental details and definitions of several types of synchronization schemes are provided. Basic concepts and preliminaries about fractional-order calculus, as well as a discussion of fractional-order chaotic systems, are also given.

In *Chapter 2*, a hybrid projective combination difference synchronization scheme is proposed for hyperchaotic complex Lü time-delay systems. Utilizing an adaptive control approach, the study employs Lyapunov stability theory to establish the stability of error states, controllers, and parameter update laws, successfully achieving synchronization between two identical hyperchaotic complex Lü time-delay systems and one slave system. In *Chapter 3*, the detailed dynamical analysis of a newly proposed 3D chaotic system is analysed. The study investigates its Lyapunov exponents, time series, and Hamilton energy to gain a deep understanding of its dynamics. Crucially, the Lagrange multiplier method is applied to solve an optimization problem analytically, calculating a precise ultimate bound set for the 3D chaotic system. The application of this derived bound set in synchronization is also discussed.

In *Chapter 4*, motivated by the emergence and practical relevance of fractional-order systems, a new 3D fractional-order system is constructed. Its dynamics are discussed through phase portraits, time series, Lyapunov exponents, and bifurcation diagrams by varying parameter values. The Hamilton energy function for the new fractional-order system is determined, revealing profound connections between system behavior and parameter variations. Additionally, the Mittag-Leffler globally attractive set and the Mittag-Leffler positive invariant set are derived for the fractional-order system. In *Chapter 5*, dynamical analysis of both integer-order and fractional-order 4D chaotic systems are analysed. The Lagrange multiplier method is again employed to calculate a precise ultimate bound set of the 4D chaotic systems. The Hamilton energy function is investigated, and the application of the bound set in locating hidden attractors and achieving synchronization is explored.

In *Chapter 6*, the theoretical framework is applied to a real-world biological system: a competitive herbivore species network (an ecological system). The dependence of the ecological system's stability on Hamilton energy is examined, and the role of competitive modes in the emergence of chaotic behavior is analysed. Ultimate bound sets of the ecological system are obtained using Lagrange optimization and analytical techniques. To control the observed chaos, one chaotic ecological system is exponentially synchronized to another under a master-slave scheme.

The insights gained from this thesis offer numerous benefits and pathways for future exploration. Understanding and controlling chaotic systems can enhance short-term forecasting and sustain stability in highly complex and unpredictable domains such as weather prediction, financial markets, and ecological systems. Furthermore, this study contributes to the development of secure communication systems, the optimization of power grids, and advancements in robotics and artificial intelligence by enabling the prediction, control, and effective utilization of chaotic behaviors.