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Name of the Scholar : **Mohd Vasiulla**

Name of the Supervisor : Prof. Quddus Khan

Name of the Co-supervisor : Dr. Mohammad Shoeb, Associate Professor, University Polytechnic, Faculty of Engineering and Technology, Jamia Millia Islamia, New Delhi-110025, India

Department : Applied Sciences and Humanities, Faculty of Engineering and Technology, Jamia Millia Islamia, New Delhi-110025, India

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ABSTRACT

Differential geometry provides a framework for studying the geometry of smooth manifolds. Ongoing research in differential geometry often involves exploring new types of manifolds, developing techniques for understanding their curvature, and applying these ideas to various mathematical and physical problems. The present thesis has been written on the same basis and efforts were made to contribute something to the subject. The results in the thesis are well motivated, therefore it might be very helpful to the mathematical society, specially to whom, who works in this direction. Further, these results can be generalized for the various other manifolds. There are some important contributions in the present Thesis and are summarized as follows:

- **Chapter 1.** This chapter is introductory; here we describe basic definitions, formulae and results which are relevant to the subsequent chapters. Although most of these results are available in standard references on the subject, nevertheless we have collected them to make the thesis self-contained.
- **Chapter 2.** In this chapter, we study mixed generalized quasi-Einstein manifolds $MG(QE)_n$. This chapter is divided into six sections. The first section is introductory one. Second section deals with results on $MG(QE)_n$ and mixed generalized quasi-constant curvature $MG(QC)_n$. It has been shown that in a Riemannian manifold a conformally flat $MG(QE)_n$ is a $MG(QC)_n$. In third section, we proved that if a W_4 -flat $MG(QE)_n$, then it is a generalized quasi-Einstein manifold $MG(QE)_n$. In the fourth section, we proved that $MG(QE)_n$ satisfying the condition $C.Ric=0$ is a $G(QE)_n$ provided $C(X,Y,\rho,\sigma)$ is non-zero at each point of the manifold. In the next section of this chapter we proved that if a conformally flat $MG(QE)_n$ with the condition $K.Ric=0$ is either a $MG(QE)_n$ or the vector fields ρ and σ are co-directional. In the last section of this chapter, we have shown the existence of $MG(QE)_n$ by constructed a non-trivial example. Results of this chapter have been published in **Jananabha**, 52(1) (2022), 182-188.

- **Chapter 3.** This chapter deals with the study of nearly quasi-Einstein manifolds $N(QE)_n$. This chapter consists of seven sections. Section 1 is introductory. Section 2 is devoted to the study of $N(QE)_n$ with the condition $K.Ric=0$. We obtained a necessary and sufficient condition $N(QE)_n$ is Ricci semi-symmetric if and only if D is semi-symmetric. In Section 3, we study $N(QE)_n$ with the condition $P.Ric=0$ and proved that $N(QE)_n$ reduces to an Einstein manifold. In Section 4, we studied $N(QE)_n$ with the condition $C.Ric=0$ and shown that $N(QE)_n$ to an Einstein manifold. In Section 5, we studied $N(QE)_n$ with the condition $N.Ric=0$ and obtained $N(QE)_n$ reduces to an Einstein manifold. In Section 6, we studied $N(QE)_n$ with the condition $W_2.Ric=0$ and proved that $N(QE)_n$ reduces to an Einstein manifold. Finally, we have constructed an example to show the existence of this manifold. Results of this chapter have been published in **Bulletin of Calcutta Mathematical Society**, 113(6) (2021), 567-576.
- **Chapter 4.** Generalized Ricci recurrent manifold have been studied in Chapter 4. It is divided in five sections. After introduction in Section 2, we obtained a sufficient condition for a generalized Ricci-recurrent manifolds admitting a unit concircular vector field and Codazzi type of Ricci tensor to be a quasi-Einstein manifold. In Section 3, we have also obtained a sufficient condition for a generalized Ricci-recurrent manifold admitting quadratic Killing Ricci tensor to be a quasi-Einstein manifold. Conformally flat generalized Ricci-recurrent manifold studied in Section 4. It has been shown that if the basic vector fields are co-directional then conformally flat generalized Ricci-recurrent manifold reduces to a quasi-Einstein manifold. Next we proved, if the basic vector fields are co-directional then conformally flat generalized Ricci-recurrent manifold reduces to a manifold of quasi-constant curvature. In Section 5 we give an example of such a manifold. Results of this chapter have been published in **J. Math. Cont. Sci. Appl.**, 5(2) (2019), 79-89.
- **Chapter 5.** In this chapter, we have introduced and studied a new kind of quasi-constant curvature called generalized super-quasi-constant curvature and have obtained interesting and fruitful results on it. This chapter has five sections. First section is introductory. In Section 2 we prove that a super quasi-umbilical hypersurface of a manifold of constant curvature is a manifold of generalized super quasi-constant curvature and also proved that the manifold of generalized super quasi-constant curvature is a super quasi-Einstein manifold. In Section 3 of this chapter deals with the example of generalized super quasi-constant curvature. In the next section, we obtained a sufficient condition for generalized super quasi-constant curvature to be a super quasi-Einstein manifold also an example of super quasi-Einstein manifold is given. Results of this chapter have been published in **Ann. Comm. Maths.**, 5(3) (2022), 194-206.
- **Chapter 6.** In this chapter, we study mixed generalized quasi-Einstein manifolds $MG(QE)_n$. This chapter contains seven sections. Section 1 is introductory level. In Section 2 we study $MG(QE)_n$ with Codazzi tensor. It has been shown that the 1-forms are closed if the Ricci tensor of this manifold is of Codazzi type. We have also proved that if the 1-forms are not parallel together, then Ricci symmetric $MG(QE)_n$ reduces

to an Einstein manifold. Section 3 is devoted to some results on mixed generalized quasi-Einstein warped product manifolds and found results on it. The fourth section dedicated to mixed generalized quasi-Einstein GRW spacetimes. It has been observed that if v^i is orthogonal to both 1-forms and λ is constant, then such manifold reduces to an Einstein manifold. In Section 5, we proved that in a mixed generalized quasi-Einstein pseudo Ricci-symmetric manifolds the scalar curvature must vanish and it has been shown that mixed generalized quasi-Einstein pseudo Ricci-symmetric manifolds with cyclic parallel Ricci tensor is Ricci flat. Finally, we have constructed two examples of this manifold. Results of this chapter have been accepted in **J. App. Maths. & Infor**, (2023).